

Edexcel A-Level Mathematics

Comprehensive Revision Notes

Specification 9MA0 · Full A-Level · Pure · Statistics · Mechanics

Includes ALL AS Topics (1-18) + ALL A-Level Additional Topics (19-30)

Full Explanations · Key Formulae · Worked Examples · Exam Tips · Formula Reference

Section 1: Pure Mathematics (Papers 1 & 2: 9MA0/01, 9MA0/02)

AS: Proof · Algebra & Functions · Coordinate Geometry · Sequences & Series ·
Trigonometry · Exponentials & Logarithms · Differentiation · Integration · Vectors
A-Level: Algebraic Methods · Functions · Advanced Trig · Parametric Equations · Further
Differentiation · Further Integration · Numerical Methods · 3D Vectors

Section 2: Statistics (Paper 3: 9MA0/31 or 9MA0/32)

AS: Sampling · Data Presentation · Probability · Binomial Distribution · Hypothesis Testing
A-Level: Normal Distribution · Regression & Correlation · Hypothesis Testing (Normal)

Section 3: Mechanics (Paper 3: 9MA0/31 or 9MA0/32)

AS: Quantities & Modelling · Kinematics · Forces & Newton's Laws · Variable Acceleration
A-Level: Moments

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Section 1

Pure Mathematics

Edexcel 8MA0/01 · Topics 1–9: Proof · Algebra · Coordinate Geometry · Sequences · Trigonometry · Calculus · Vectors

Topic 1: Proof

1.1 Types of Mathematical Proof

Proof: A rigorous logical argument showing that a mathematical statement is always true, using definitions, previously proved results, and logical steps.

Method	Description	When to use
Proof by deduction	Start from known facts/axioms and derive the result using logical steps	General algebraic/numerical statements
Proof by exhaustion	Check all possible cases — only practical when there are finitely many cases	Small finite sets of values
Proof by counter-example	Find one example where the statement is FALSE — this disproves it	Disproving 'for all' statements
Proof by contradiction	Assume the statement is FALSE and derive a logical contradiction	Proving irrational numbers; infinity of primes

Worked Example: Proof by deduction

Prove: the sum of any two odd numbers is even.

Proof: Let the two odd numbers be $(2m + 1)$ and $(2n + 1)$ where m, n are integers.

Their sum = $(2m + 1) + (2n + 1) = 2m + 2n + 2 = 2(m + n + 1)$.

Since $m + n + 1$ is an integer, the sum is even. **QED**

Worked Example: Proof by counter-example

Disprove: ' $n^2 + n + 41$ is prime for all positive integers n .'

Let $n = 41$: $n^2 + n + 41 = 41^2 + 41 + 41 = 41(41 + 1 + 1) = 41 \times 43$, which is not prime.

Counter-example found — statement is FALSE.

Worked Example: Proof by contradiction

Prove: $\sqrt{2}$ is irrational.

Assume $\sqrt{2} = p/q$ in lowest terms (p, q integers, no common factor).

Then $2 = p^2/q^2$, so $p^2 = 2q^2 \rightarrow p^2$ is even $\rightarrow p$ is even $\rightarrow p = 2k$.

So $4k^2 = 2q^2 \rightarrow q^2 = 2k^2 \rightarrow q$ is even.

But p and q both even contradicts 'no common factor'. Contradiction — $\sqrt{2}$ is irrational. **QED**

★ **Exam Tip:** Always conclude your proof with QED (quod erat demonstrandum) or $\square$. For AS, make sure each step clearly follows from the previous one using algebra or definitions.

Topic 2: Algebra and Functions

2.1 Laws of Indices

Law	Rule	Example
Multiplication	$a^m \times a^n = a^{m+n}$	$x^3 \times x^4 = x^7$
Division	$a^m \div a^n = a^{m-n}$	$x^5 \div x^2 = x^3$
Power of a power	$(a^m)^n = a^{mn}$	$(x^2)^3 = x^6$
Zero index	$a^0 = 1$ ($a \neq 0$)	$5^0 = 1$
Negative index	$a^{-n} = 1/a^n$	$x^{-2} = 1/x^2$
Fractional index	$a^{1/n} = \sqrt[n]{a}$ (nth root)	$8^{1/3} = \sqrt[3]{8} = 2$
Combined fractional	$a^{m/n} = (\sqrt[n]{a})^m$	$27^{2/3} = (\sqrt[3]{27})^2 = 9$

★ **Exam Tip:** Always simplify index expressions fully. Watch for $a^{-1/2} = 1/\sqrt{a}$ and mixed fractional-negative indices.

2.2 Surds

Surd: An irrational number expressed as a root that cannot be simplified to a rational number. E.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $2\sqrt{7}$. Note: $\sqrt{4} = 2$ is NOT a surd.

Surd rules: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and $\sqrt{a/b} = \sqrt{a} / \sqrt{b}$

Simplifying surds: $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Expanding: $(3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 2 = 7$ (difference of two squares)

Rationalising the denominator: Removing surds from the denominator of a fraction by multiplying numerator and denominator by the conjugate (or surd itself).

Worked Example: Rationalising the denominator

Case 1 — denominator is $\sqrt{a}$: $3/\sqrt{5} = (3 \times \sqrt{5})/(\sqrt{5} \times \sqrt{5}) = 3\sqrt{5}/5$

Case 2 — denominator is $a + \sqrt{b}$: $4/(3 + \sqrt{2}) = 4(3 - \sqrt{2})/((3 + \sqrt{2})(3 - \sqrt{2})) = 4(3 - \sqrt{2})/(9 - 2) = 4(3 - \sqrt{2})/7$

2.3 Quadratic Equations

Quadratic equation: An equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$. Has at most 2 real solutions (roots).

Methods of solution:

- **Factorising:** write $ax^2 + bx + c = a(x - p)(x - q)$ where p, q are the roots. Works when roots are rational.
- e.g. $x^2 - 5x + 6 = (x - 2)(x - 3) = 0 \rightarrow x = 2$ or $x = 3$

- **Completing the square:** rewrite as $a(x + p)^2 + q$. Used to find vertex of parabola and solve.
- e.g. $x^2 + 6x + 5$: $x^2 + 6x = (x+3)^2 - 9$, so $(x+3)^2 - 9 + 5 = (x+3)^2 - 4 = 0$
- **Quadratic formula:** always works.

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ [for $ax^2 + bx + c = 0$]

Discriminant: $\Delta = b^2 - 4ac$ [$\Delta > 0$: two real roots | $\Delta = 0$: one repeated root | $\Delta < 0$: no real roots]

Completing the square — vertex form:

Vertex form: $ax^2 + bx + c = a(x + b/2a)^2 + (c - b^2/4a)$ [Vertex at $(-b/2a, c - b^2/4a)$]

★ **Exam Tip:** Learn to complete the square quickly: $x^2 + bx = (x + b/2)^2 - (b/2)^2$. The vertex form immediately gives the minimum/maximum point and the axis of symmetry.

2.4 Simultaneous Equations

Solving two equations in two unknowns. At AS level: linear/linear (substitution or elimination) and linear/quadratic (substitution).

Worked Example: Linear/quadratic simultaneous equations

Solve: $y = 2x + 1$ and $x^2 + y^2 = 10$

Substitute $y = 2x + 1$ into $x^2 + y^2 = 10$:

$$x^2 + (2x+1)^2 = 10 \rightarrow x^2 + 4x^2 + 4x + 1 = 10 \rightarrow 5x^2 + 4x - 9 = 0$$

Factorise: $(5x + 9)(x - 1) = 0 \rightarrow x = -9/5$ or $x = 1$

When $x = 1$: $y = 3$. When $x = -9/5$: $y = 2(-9/5) + 1 = -13/5$

2.5 Inequalities

Linear inequalities: solve like equations but **reverse the inequality sign when multiplying or dividing by a negative number**.

Quadratic inequalities — use a sketch of the parabola:

Worked Example: Quadratic inequality

Solve $x^2 - 3x - 10 > 0$

Factorise: $(x - 5)(x + 2) > 0$

Roots at $x = 5$ and $x = -2$. Parabola opens upward (positive x^2 coefficient).

Parabola > 0 outside the roots: $x < -2$ or $x > 5$

Sign diagram / sketch method: always sketch the quadratic to identify where it is positive or negative. For $ax^2 + bx + c \geq 0$ ($a > 0$): answer is OUTSIDE or ON the roots. For $ax^2 + bx + c \leq 0$ ($a > 0$): answer is BETWEEN the roots.

★ **Exam Tip:** NEVER multiply both sides of an inequality by x (or any expression) unless you know the sign. Instead, bring everything to one side and factorise.

2.6 Polynomials — Factor and Remainder Theorems

Factor Theorem: $(x - a)$ is a factor of $f(x) \iff f(a) = 0$ [Used to find linear factors of polynomials]

Remainder Theorem: When $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$ [No need to perform long division to find the remainder]

Worked Example: Using the Factor Theorem

Show that $(x - 2)$ is a factor of $f(x) = x^3 - 3x^2 + 4$, then fully factorise.

$f(2) = 8 - 12 + 4 = 0 \checkmark \rightarrow (x - 2)$ is a factor.

Divide: $x^3 - 3x^2 + 4 = (x - 2)(x^2 - x - 2) = (x - 2)(x - 2)(x + 1) = (x - 2)^2(x + 1)$

Polynomial long division or inspection can be used to find the quotient once a factor is known. Practice both methods — inspection (comparing coefficients) is often faster.

Topic 3: Coordinate Geometry in the (x, y) Plane

3.1 Straight Lines

Concept	Formula	Notes
Gradient	$m = (y_2 - y_1)/(x_2 - x_1)$	Rise over run between two points
Equation (slope-intercept)	$y = mx + c$	m = gradient, c = y-intercept
Equation (point-gradient)	$y - y_1 = m(x - x_1)$	Useful when given a point and gradient
General form	$ax + by + c = 0$	Rearrange to find gradient: $m = -a/b$
Distance between two points	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Pythagoras in 2D
Midpoint	$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$	Average of coordinates

Parallel lines: Same gradient: $m_1 = m_2$

Perpendicular lines: Gradients multiply to -1 : $m_1 \times m_2 = -1$ (i.e. $m_2 = -1/m_1$)

[Negative reciprocal]

Worked Example: Finding the equation of a perpendicular bisector

Find the perpendicular bisector of the line segment joining A(2, 1) and B(6, 5).

Gradient of AB: $m = (5-1)/(6-2) = 4/4 = 1$

Gradient of perpendicular: $m_{\perp} = -1/1 = -1$

Midpoint M = $((2+6)/2, (1+5)/2) = (4, 3)$

Equation: $y - 3 = -1(x - 4) \rightarrow y = -x + 7$ or $x + y = 7$

3.2 Circles

Equation of a circle (centre-radius form): $(x - a)^2 + (y - b)^2 = r^2$ [Centre (a, b), radius r]

Expanded form: $x^2 + y^2 + 2fx + 2gy + c = 0$ [Centre $(-f, -g)$, radius = $\sqrt{f^2 + g^2 - c}$]

Key circle properties (AS level):

- The angle in a semicircle is 90° (angle subtended by diameter).
- The tangent to a circle is perpendicular to the radius at the point of contact.
- Perpendicular from centre to a chord bisects the chord.

Worked Example: Finding the equation of a circle and tangent

A circle has diameter with endpoints P(1, 3) and Q(7, 11). Find the equation.

Centre = midpoint = $((1+7)/2, (3+11)/2) = (4, 7)$

Radius = $\frac{1}{2} \times \text{distance PQ} = \frac{1}{2} \times \sqrt{((7-1)^2 + (11-3)^2)} = \frac{1}{2} \times \sqrt{(36+64)} = \frac{1}{2} \times 10 = 5$

Equation: $(x - 4)^2 + (y - 7)^2 = 25$

Find tangent at P(1, 3):

Gradient of radius CP: $(3-7)/(1-4) = (-4)/(-3) = 4/3$

Tangent gradient = $-3/4$ (perpendicular to radius)

Tangent: $y - 3 = -3/4(x - 1) \rightarrow 4y - 12 = -3x + 3 \rightarrow 3x + 4y = 15$

To convert from expanded to centre-radius form: complete the square on x and y separately.

Worked Example: Completing the square on a circle equation

Write $x^2 + y^2 - 6x + 4y - 3 = 0$ in centre-radius form.

$(x^2 - 6x) + (y^2 + 4y) = 3$

$(x - 3)^2 - 9 + (y + 2)^2 - 4 = 3$

$(x - 3)^2 + (y + 2)^2 = 16 \rightarrow \text{Centre } (3, -2), \text{ radius } 4$

★ **Exam Tip:** For intersection of a line and circle: substitute the line into the circle equation and use the discriminant to determine whether they intersect ($\Delta > 0$), are tangent ($\Delta = 0$), or don't meet ($\Delta < 0$).

Topic 4: Sequences and Series

4.1 Arithmetic Sequences (APs)

Arithmetic sequence: A sequence where consecutive terms differ by a constant value called the **common difference** d . General term: $a, a+d, a+2d, \dots$

Formula	Expression	Meaning
n-th term	$u_n = a + (n - 1)d$	a = first term, d = common difference, n = term number
Sum of n terms	$S_n = n/2 \times (2a + (n-1)d)$	Sum of first n terms of an AP
Alternative sum	$S_n = n/2 \times (a + l)$	l = last term; use when first and last terms known
Condition for AP	$d = u_{n+1} - u_n = \text{constant}$	Check consecutive differences are equal

Worked Example: AP problem

An AP has 3rd term 11 and 7th term 27. Find a and d , then S_{15} .

$$u_3 = a + 2d = 11 \text{ and } u_7 = a + 6d = 27$$

Subtract: $4d = 16 \rightarrow d = 4$, then $a = 11 - 8 = 3$

$$S_{15} = 15/2 \times (2(3) + 14(4)) = 15/2 \times (6 + 56) = 15/2 \times 62 = 465$$

4.2 Geometric Sequences (GPs)

Geometric sequence: A sequence where each term is multiplied by a constant called the **common ratio** r . General term: $a, ar, ar^2, ar^3, \dots$

Formula	Expression	Notes
n-th term	$u_n = ar^{n-1}$	a = first term, r = common ratio
Sum of n terms	$S_n = a(1 - r^n) / (1 - r)$	Valid for $r \neq 1$; equivalent: $a(r^n - 1)/(r - 1)$
Sum to infinity	$S_\infty = a / (1 - r)$	Only valid when $ r < 1$ (convergent GP)
Common ratio	$r = u_{n+1} / u_n = \text{constant}$	Check by dividing consecutive terms

Convergent GP condition: $|r| < 1 \rightarrow \text{sum to infinity exists}$ [If $|r| \geq 1$ the series diverges]

Worked Example: GP sum to infinity

A GP has first term 8 and common ratio $1/3$. Find the sum to infinity.

$|r| = 1/3 < 1 \checkmark \rightarrow \text{convergent.}$

$$S_\infty = 8 / (1 - 1/3) = 8 / (2/3) = 8 \times 3/2 = 12$$

4.3 Binomial Expansion

Binomial expansion: The expansion of $(a + b)^n$ for a positive integer n . The coefficients are given by Pascal's triangle or binomial coefficients.

Binomial Theorem: $(a + b)^n = \sum {}^nC_r \times a^{n-r} \times b^r$ for $r = 0$ to n [where ${}^nC_r = n! / (r!(n-r)!)$]

Binomial coefficient: ${}^nC_r = n! / (r!(n-r)!)$ [Also written $C(n,r)$ or $(n \text{ choose } r)$; found on calculator]

Worked Example: Binomial expansion

Expand $(2 + x)^4$.
 $= {}^4C_0(2)^4(x)^0 + {}^4C_1(2)^3(x)^1 + {}^4C_2(2)^2(x)^2 + {}^4C_3(2)^1(x)^3 + {}^4C_4(2)^0(x)^4$
 $= 1 \times 16 \times 1 + 4 \times 8 \times x + 6 \times 4 \times x^2 + 4 \times 2 \times x^3 + 1 \times 1 \times x^4$
 $= 16 + 32x + 24x^2 + 8x^3 + x^4$

Worked Example: Finding a specific term

Find the coefficient of x^3 in $(3 - 2x)^5$.
General term: ${}^5C_r \times (3)^{5-r} \times (-2x)^r$
For x^3 : $r = 3 \rightarrow {}^5C_3 \times 3^2 \times (-2)^3 \times x^3 = 10 \times 9 \times (-8) \times x^3 = -720x^3$
Coefficient of x^3 is **-720**

★ **Exam Tip:** ${}^nC_r = {}^nC_{(n-r)}$ by symmetry — so ${}^5C_4 = {}^5C_1 = 5$. Also, ${}^nC_0 = {}^nC_n = 1$ always. For $(1 + bx)^n$: the expansion is $1 + nbx + n(n-1)/2! \times (bx)^2 + \dots$

Topic 5: Trigonometry

5.1 Basic Trigonometry — SOHCAHTOA

Ratio	Definition	Memory aid
$\sin \theta$	Opposite / Hypotenuse	SOH
$\cos \theta$	Adjacent / Hypotenuse	CAH
$\tan \theta$	Opposite / Adjacent = $\sin \theta / \cos \theta$	TOA

Exact values (learn these):

θ (degrees)	θ (radians)	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3} = \sqrt{3}/3$
45°	$\pi/4$	$1/\sqrt{2} = \sqrt{2}/2$	$1/\sqrt{2} = \sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	undefined

5.2 Sine Rule and Cosine Rule

Sine Rule: $a/\sin A = b/\sin B = c/\sin C$ [Use when: two angles and a side (AAS/ASA), or two sides and a non-included angle (SSA — ambiguous case)]

Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$ [Use when: two sides and included angle (SAS), or all three sides (SSS) to find an angle]

Area of triangle: $\text{Area} = \frac{1}{2} ab \sin C$ [Use when: two sides and the included angle are known]

For the ambiguous case (SSA): when $\sin A = \sin(180^\circ - A)$, there may be two possible triangles. Always check whether a second solution is geometrically valid.

★ **Exam Tip:** Draw a diagram for every triangle problem. Label sides (a, b, c) opposite to their angles (A, B, C). Check: angles in a triangle sum to 180° .

5.3 Radians, Arc Length and Sector Area

Radian definition: $2\pi \text{ radians} = 360^\circ \rightarrow 1 \text{ radian} \approx 57.3^\circ$ [Conversion: degrees to radians: multiply by $\pi/180$; radians to degrees: multiply by $180/\pi$]

Formula	Expression	Where
Arc length	$l = r\theta$	r = radius, θ in radians
Area of sector	$A = \frac{1}{2}r^2\theta$	θ in radians

Area of segment	$A = \frac{1}{2}r^2(\theta - \sin \theta)$	Sector area minus triangle area
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★ **Exam Tip:** Radians must be used in arc length and sector area formulas. A common error is to leave θ in degrees — always convert first.

5.4 Graphs of Trigonometric Functions

Function	Period	Key features	Graph shape
$y = \sin x$	$360^\circ (2\pi)$	Range: $-1 \leq y \leq 1$; $\sin 0^\circ = 0$; passes through origin	Smooth wave starting at (0,0) rising to $(90^\circ, 1)$
$y = \cos x$	$360^\circ (2\pi)$	Range: $-1 \leq y \leq 1$; $\cos 0^\circ = 1$; y-intercept = 1	Smooth wave starting at (0,1) decreasing to $(90^\circ, 0)$
$y = \tan x$	$180^\circ (\pi)$	Range: all reals; undefined at $\pm 90^\circ, \pm 270^\circ$; passes through origin	Repeated S-curves with vertical asymptotes at $90^\circ + 180^\circ n$

Transformations of trig graphs:

- $y = a \sin(bx + c) + d$: amplitude $|a|$, period $360^\circ/b$ (or $2\pi/b$), phase shift $-c/b$, vertical shift d .
- Vertical stretch by factor a : $y = a \sin x$ — amplitude changes.
- Horizontal stretch by factor $1/b$: $y = \sin(bx)$ — period changes to $360^\circ/b$.

5.5 Trigonometric Identities

Pythagorean identity: $\sin^2\theta + \cos^2\theta = 1$ [Always true for any angle θ]
Tangent identity: $\tan \theta = \sin \theta / \cos \theta$ [Derived from SOHCAHTOA]

These identities are used to simplify expressions and solve equations. Common rearrangements:

- $\sin^2\theta = 1 - \cos^2\theta$ or $\cos^2\theta = 1 - \sin^2\theta$
- Replace $\sin^2\theta$ or $\cos^2\theta$ to get a quadratic in one trig function.

5.6 Solving Trigonometric Equations

To solve trig equations in a given interval, use the following steps:

- **Step 1:** Rearrange to $\sin \theta = k$, $\cos \theta = k$, or $\tan \theta = k$ (or a quadratic in trig function).
- **Step 2:** Find the principal value using inverse trig ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$).
- **Step 3:** Find all solutions in the given interval using symmetry and periodicity.

Function	Symmetry rule	Period
$\sin \theta = k$	Second solution: $180^\circ - \theta$	Repeats every 360°
$\cos \theta = k$	Second solution: $360^\circ - \theta$ (or $-\theta$)	Repeats every 360°
$\tan \theta = k$	Second solution: $\theta + 180^\circ$	Repeats every 180°

Worked Example: Solving a trig equation

Solve $2\sin^2 x - \sin x - 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.

Let $s = \sin x$: $2s^2 - s - 1 = 0 \rightarrow (2s + 1)(s - 1) = 0$

$s = -1/2$ or $s = 1$

$\sin x = 1$: $x = 90^\circ$

$\sin x = -1/2$: principal value $= -30^\circ$; solutions in $[0^\circ, 360^\circ]$: $x = 180^\circ + 30^\circ = 210^\circ$ and $x = 360^\circ - 30^\circ = 330^\circ$

Solutions: $x = 90^\circ, 210^\circ, 330^\circ$

★ **Exam Tip:** CAST diagram: All (0° – 90°), Sine (90° – 180°), Tangent (180° – 270°), Cosine (270° – 360°). Use this to identify which quadrants the solutions lie in.

Topic 6: Exponentials and Logarithms

6.1 Exponential Functions

Exponential function: A function of the form $f(x) = a^x$ where $a > 0$, $a \neq 1$. Key properties: always positive; passes through $(0, 1)$; domain all reals; range > 0 .

The most important base is **$e \approx 2.71828...$** — Euler's number. The function $y = e^x$ is the natural exponential function and its own derivative (the rate of change of e^x equals e^x).

Base	Function	Growth/decay	Graph
$a > 1$	$y = a^x$	Growth — increases as x increases	Passes $(0,1)$; rises steeply for $x > 0$; approaches 0 for $x \rightarrow -\infty$
$0 < a < 1$	$y = a^x$	Decay — decreases as x increases	Passes $(0,1)$; falls toward 0 for $x > 0$; rises steeply for $x \rightarrow -\infty$
$a = e$	$y = e^x$	Natural exponential growth	Same shape as $a > 1$; tangent at $(0,1)$ has gradient exactly 1

Transformations: $y = Ae^{kx}$ is a stretch. $y = e^{-x}$ is the reflection of e^x in the y -axis (decay). $y = e^x + k$ is a vertical translation.

6.2 Logarithms

Logarithm: $\log_a(x) = y$ means $a^y = x$. The logarithm is the inverse of exponentiation. $\log_a(x)$ asks: 'to what power must I raise a to get x ?'

Natural logarithm: $\ln x = \log_e(x) \leftrightarrow e^y = x$ [$\ln$ and e are inverse functions: $\ln(e^x) = x$ and $e^{(\ln x)} = x$]

Law of logarithms	Formula	Example
Product rule	$\log_a(xy) = \log_a(x) + \log_a(y)$	$\log(6) = \log(2) + \log(3)$
Quotient rule	$\log_a(x/y) = \log_a(x) - \log_a(y)$	$\log(3) = \log(6) - \log(2)$
Power rule	$\log_a(x^n) = n \log_a(x)$	$\log(8) = 3 \log(2)$
Log of 1	$\log_a(1) = 0$	Because $a^0 = 1$
Log of base	$\log_a(a) = 1$	Because $a^1 = a$
Change of base	$\log_a(x) = \log_b(x) / \log_b(a)$	$\log_3(7) = \ln 7 / \ln 3$ (use calculator)

Worked Example: Solving exponential and log equations

(a) Solve $3e^x = 15$: $e^x = 5 \rightarrow x = \ln 5 \approx 1.609$

(b) Solve $\log_2(x) + \log_2(x - 2) = 3$:

$\log_2(x(x-2)) = 3 \rightarrow x(x-2) = 2^3 = 8 \rightarrow x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0 \rightarrow x = 4$ or $x = -2$

Check: $\log_2(-2)$ undefined $\rightarrow x = 4$ only

6.3 Exponential Modelling

Many real-world quantities grow or decay exponentially: population growth, radioactive decay, cooling, compound interest.

Exponential growth/decay model: $y = Ae^{kt}$ or $y = Aa^t$ [A = initial value; $k > 0$ growth, $k < 0$ decay; t = time]

To linearise: take $\ln$ of both sides. If $y = Ae^{kx}$ then $\ln y = \ln A + kx$ — plotting $\ln y$ against x gives a straight line with gradient k and y -intercept $\ln A$.

★ **Exam Tip:** Log laws are the most frequently tested topic in this section. Practise simplifying expressions and solving equations step by step. Remember: you can only combine logs with the SAME base.

Topic 7: Differentiation

7.1 The Derivative from First Principles

Derivative: The gradient of the tangent to a curve at a point. Measures the instantaneous rate of change. Notation: dy/dx , $f'(x)$, or $\dot{y}$.

First principles definition: $f'(x) = \lim_{h \rightarrow 0} (f(x+h) - f(x)) / h$ [The limit of the gradient of a chord as it approaches the tangent]

Worked Example: Differentiation from first principles

Prove that $d/dx(x^2) = 2x$ from first principles.

$$f(x) = x^2, f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$$

$$f'(x) = \lim_{h \rightarrow 0} (x^2 + 2xh + h^2 - x^2)/h = \lim_{h \rightarrow 0} (2xh + h^2)/h = \lim_{h \rightarrow 0} (2x + h) = 2x \checkmark$$

7.2 Differentiating Polynomials and Standard Functions

Function	Derivative	Notes
x^n	nx^{n-1}	Power rule — multiply by power, reduce power by 1
ax^n	anx^{n-1}	Constant multiple rule
$f(x) + g(x)$	$f'(x) + g'(x)$	Sum rule — differentiate term by term
constant k	0	Gradient of a constant function is zero
e^x	e^x	Exponential function is its own derivative
e^{kx}	ke^{kx}	Chain rule applied
$\ln x$	$1/x$	$x > 0$
$\sin x$	$\cos x$	x in radians
$\cos x$	$-\sin x$	x in radians
a^x	$a^x \ln a$	Exponential with arbitrary base

7.3 Chain Rule, Product Rule, Quotient Rule

Chain Rule: $dy/dx = (dy/du) \times (du/dx)$ for $y = f(g(x))$ [Differentiate outer function $\times$ derivative of inner function]

Worked Example: Chain rule

Differentiate $y = (3x^2 + 1)^5$.

Let $u = 3x^2 + 1$, $y = u^5$.

$$dy/du = 5u^4, du/dx = 6x$$

$$dy/dx = 5u^4 \times 6x = 30x(3x^2 + 1)^4$$

Product Rule: $\frac{d}{dx}[uv] = u(\frac{dv}{dx}) + v(\frac{du}{dx}) = uv' + vu'$ [Use when two functions are multiplied]

Worked Example: Product rule

Differentiate $y = x^2 e^x$.

$u = x^2, v = e^x \rightarrow u' = 2x, v' = e^x$

$\frac{dy}{dx} = x^2 e^x + e^x(2x) = e^x(x^2 + 2x) = xe^x(x + 2)$

Quotient Rule: $\frac{d}{dx}[u/v] = \frac{(v(\frac{du}{dx}) - u(\frac{dv}{dx}))}{v^2} = \frac{(vu' - uv')}{v^2}$ [Use when one function is divided by another]

Worked Example: Quotient rule

Differentiate $y = \sin x / x$.

$u = \sin x, v = x \rightarrow u' = \cos x, v' = 1$

$\frac{dy}{dx} = \frac{(x \cos x - \sin x \times 1)}{x^2} = \frac{(x \cos x - \sin x)}{x^2}$

7.4 Applications of Differentiation

Tangents and Normals:

- The gradient of the tangent at $x = a$ is $f'(a)$.
- Tangent equation: $y - f(a) = f'(a)(x - a)$
- Normal is perpendicular to tangent: gradient of normal = $-1/f'(a)$

Increasing/Decreasing Functions and Stationary Points:

Stationary points: $f'(x) = 0$ at a stationary point [Could be maximum, minimum, or point of inflection]

Test	Local maximum	Local minimum	Point of inflection
Second derivative test	$f''(x) < 0$	$f''(x) > 0$	$f''(x) = 0$ (check further)
First derivative sign change	f' changes + to -	f' changes - to +	f' doesn't change sign

Worked Example: Finding stationary points

Find and classify stationary points of $f(x) = x^3 - 6x^2 + 9x + 2$.

$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3) = 0$

Stationary points at $x = 1$ and $x = 3$.

$f''(x) = 6x - 12$

At $x = 1$: $f''(1) = -6 < 0 \rightarrow$ **local maximum**; $f(1) = 1 - 6 + 9 + 2 = 6 \rightarrow$ point (1, 6)

At $x = 3$: $f''(3) = 6 > 0 \rightarrow$ **local minimum**; $f(3) = 27 - 54 + 27 + 2 = 2 \rightarrow$ point (3, 2)

Optimisation:

Find the maximum or minimum value of a quantity by setting the derivative to zero and solving. Always verify the nature using the second derivative or checking nearby values. Check endpoints if the domain is restricted.

★ **Exam Tip:** Show your derivative equals zero AND state the nature (max/min) of the point. In 'show that' questions, every step of algebra must be shown explicitly.

Topic 8: Integration

8.1 Integration as Reverse Differentiation

Indefinite integral: $\int f(x) dx = F(x) + C$ where $F'(x) = f(x)$. C is the constant of integration — always include it.

Function $f(x)$	$\int f(x) dx$	Notes
x^n ($n \neq -1$)	$x^{n+1}/(n+1) + C$	Increase power by 1, divide by new power
ax^n	$ax^{n+1}/(n+1) + C$	Constant multiple rule
e^x	$e^x + C$	
e^{kx}	$e^{kx}/k + C$	Divide by the coefficient of x
$1/x$ ($x > 0$)	$\ln x + C$	
$\sin x$	$-\cos x + C$	x in radians
$\cos x$	$\sin x + C$	x in radians
$\sin(kx)$	$-\cos(kx)/k + C$	
$\cos(kx)$	$\sin(kx)/k + C$	

Worked Example: Indefinite integration

Find $\int (3x^2 - 4x + 5) dx$.

$$= 3(x^3/3) - 4(x^2/2) + 5x + C = x^3 - 2x^2 + 5x + C$$

8.2 Integration by Substitution and Reverse Chain Rule

For integrals of the form $\int f(g(x))g'(x) dx$ — look for a function and its derivative.

Worked Example: Reverse chain rule

Find $\int 6x(3x^2 + 1)^4 dx$.

Notice: derivative of $(3x^2 + 1)$ is $6x$ — this is the reverse chain rule.

$$\int 6x(3x^2 + 1)^4 dx = (3x^2 + 1)^5 / 5 + C$$

Worked Example: Integration by substitution

Find $\int x\sqrt{x^2 + 1} dx$ using substitution $u = x^2 + 1$.

$$du/dx = 2x \rightarrow du = 2x dx \rightarrow x dx = du/2$$

$$\int x\sqrt{x^2+1} dx = \int \sqrt{u} \times du/2 = \frac{1}{2} \int u^{1/2} du = \frac{1}{2} \times (2/3)u^{3/2} + C = (1/3)(x^2+1)^{3/2} + C$$

8.3 Definite Integrals and Areas

Definite integral: $\int [a \text{ to } b] f(x) dx = [F(x)]_a^b = F(b) - F(a)$ [No constant of integration needed]

Area under a curve: $A = \int[a \text{ to } b] |f(x)| \, dx$ [Use modulus — areas are always positive. Split at x-intercepts if curve crosses x-axis.]

Area between two curves: $A = \int[a \text{ to } b] (f(x) - g(x)) \, dx$ [where $f(x) \geq g(x)$ throughout $[a, b]$. Find intersection points as limits.]

Worked Example: Area under a curve

Find the area enclosed between $y = x^2 - x - 2$ and the x-axis.

Find roots: $x^2 - x - 2 = (x-2)(x+1) = 0 \rightarrow x = -1$ and $x = 2$

Curve is BELOW x-axis between $x = -1$ and $x = 2$ (check: $f(0) = -2 < 0$)

Area = $|\int[-1 \text{ to } 2] (x^2 - x - 2) \, dx| = |[x^3/3 - x^2/2 - 2x]_{-1}^2|$

$= |(8/3 - 2 - 4) - (-1/3 - 1/2 + 2)| = |(8/3 - 6) - (-5/6 + 2)| = |-10/3 - 7/6| = |-27/6| = 9/2$

Trapezium rule (numerical integration): estimates area when exact integration is hard.

Trapezium rule: $\int[a \text{ to } b] f(x) \, dx \approx h/2 \times [y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n]$ [$h = (b-a)/n$ is the strip width; n equal strips; y values at $x_0, x_1, \dots, x_n$]

★ **Exam Tip:** For areas BELOW the x-axis, $\int$ gives a negative value — take the absolute value. For areas between two curves, always check which curve is on top by testing a point in the interval.

Topic 9: Vectors

9.1 Vector Notation and Representation

Vector: A quantity with both magnitude (size) AND direction. Written as bold **a**, or with an arrow: $\rightarrow AB$, or as a column vector.

Notation	Meaning	Example
Column vector	$\mathbf{a} = (p, q)$ — component form	$(3, -2)$ means 3 in x-direction, -2 in y-direction
Unit vectors $\mathbf{i}, \mathbf{j}$	$\mathbf{i} = (1, 0), \mathbf{j} = (0, 1)$ — basis vectors	$3\mathbf{i} - 2\mathbf{j}$ is the same as $(3, -2)$
Position vector	Vector from origin O to point P: OP or $\mathbf{p}$	Position of P relative to origin
Displacement vector	Vector from A to B: $\rightarrow AB = \mathbf{b} - \mathbf{a}$ (position vectors)	Direction and distance from A to B

9.2 Vector Operations

Operation	Rule	Geometric meaning
Addition	$\mathbf{a} + \mathbf{b}$: add corresponding components	Triangle law / parallelogram law
Subtraction	$\mathbf{a} - \mathbf{b}$: subtract components	Find vector from B to A
Scalar multiple	$k\mathbf{a}$: multiply each component by k	Scales the vector; same direction ($k > 0$) or opposite ($k < 0$)
Magnitude	$ \mathbf{a} = \sqrt{x^2 + y^2}$	Length of the vector — Pythagoras in 2D
Unit vector	$\hat{\mathbf{a}} = \mathbf{a}/ \mathbf{a} $	Vector in same direction with length 1

Worked Example: Vector operations

Given $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j}$:

$$\mathbf{a} + \mathbf{b} = 4\mathbf{i} + 2\mathbf{j}$$

$$2\mathbf{a} - \mathbf{b} = 6\mathbf{i} + 8\mathbf{j} - \mathbf{i} + 2\mathbf{j} = 5\mathbf{i} + 10\mathbf{j}$$

$$|\mathbf{a}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\text{Unit vector in direction of } \mathbf{a}: \hat{\mathbf{a}} = (3/5)\mathbf{i} + (4/5)\mathbf{j}$$

9.3 Position Vectors and Geometry

Displacement $\rightarrow AB$: $\rightarrow AB = \mathbf{OB} - \mathbf{OA} = \mathbf{b} - \mathbf{a}$ [Vector FROM A TO B equals position vector of B minus position vector of A]

Midpoint M of AB: $\mathbf{OM} = (\mathbf{a} + \mathbf{b})/2$ [Average of position vectors gives midpoint position vector]

Worked Example: Geometry with vectors

Points A, B, C have position vectors $\mathbf{a} = (1, 2)$, $\mathbf{b} = (5, 4)$, $\mathbf{c} = (3, 6)$.

$\rightarrow\text{AB} = \mathbf{b} - \mathbf{a} = (4, 2)$; $\rightarrow\text{AC} = \mathbf{c} - \mathbf{a} = (2, 4)$

Note: $\rightarrow\text{AC} = (1/2)(4, 8) \neq k\rightarrow\text{AB}$, so A, B, C are not collinear.

Midpoint M of BC: $\mathbf{m} = ((5+3)/2, (4+6)/2) = (4, 5)$

Collinear points: A, B, C are collinear if $\rightarrow\text{AB} = k\rightarrow\text{AC}$ for some scalar k . This means the vectors are parallel and share point A.

★ **Exam Tip:** Vectors in AS Maths are 2D only. For 'show that points are collinear': prove one displacement vector is a scalar multiple of another. Always show the scalar explicitly.

Section 2

Statistics

Edexcel 8MA0/02 · Topics 10–14: Sampling · Data · Probability · Binomial Distribution · Hypothesis Testing

Topic 10: Statistical Sampling

10.1 Populations and Samples

Population: The complete set of items being studied. Can be finite or infinite.

Sample: A subset of the population selected for study. Should be representative to allow valid inferences.

Census: Data collected from the ENTIRE population. Accurate but expensive, time-consuming, may be impractical.

Feature	Census	Sample
Accuracy	Complete — no sampling error	Estimates may have sampling error
Cost/time	High — covers all members	Lower — smaller data set
Practicality	Impractical for large/infinite populations	Feasible for most investigations
Suitability	When population is small or data vital to each member	When population is large or testing is destructive

10.2 Sampling Methods

Method	Process	Advantage	Disadvantage
Simple random sampling	Each member equally likely to be chosen; use random numbers or lottery	Unbiased; each member has equal chance	Requires a complete sampling frame; impractical for large populations
Systematic sampling	Select every k-th member after a random start ($k = \text{population/sample size}$)	Simple to carry out; evenly spread	Sampling frame needed; may miss patterns if data is periodic

Stratified sampling	Divide population into strata (subgroups); sample proportionally from each stratum	Represents all subgroups; reduces bias	Need to know composition of population; complex
Opportunity (convenience) sampling	Select whoever is available/convenient	Quick and cheap; no sampling frame needed	Highly biased; not representative
Cluster sampling	Divide into clusters; randomly select whole clusters	Practical when population widely spread; cheap	Can be biased if clusters not representative

Sampling frame: A complete list of all members of the population from which the sample is drawn. Required for random and systematic sampling.

Sampling bias: Systematic distortion in sample selection that makes the sample unrepresentative of the population.

★ **Exam Tip:** In an exam, for stratified sampling: calculate the number from each stratum as $(\text{stratum size} / \text{total population}) \times \text{sample size}$. Always round appropriately.

Topic 11: Data Presentation and Interpretation

11.1 Types of Data

Type	Description	Examples
Qualitative (categorical)	Non-numerical data — names or categories	Colour, gender, type of car
Quantitative — discrete	Numerical; can only take specific values (usually whole numbers)	Number of children; shoe size; goals scored
Quantitative — continuous	Numerical; can take any value in a range	Height; weight; temperature; time

11.2 Measures of Location (Averages)

Measure	Definition	Best used when
Mean ($\bar{x}$)	Sum of all values / number of values = $\Sigma x/n$ (or $\Sigma fx/\Sigma f$ for frequency data)	Data is symmetric; no extreme outliers; interval/ratio data
Median	Middle value when data is ordered; average of two middle values for even n	Data is skewed or has outliers; ordinal data
Mode	Most frequently occurring value (can have none, one, or several)	Categorical data; identifying most common value

Mean from frequency table: $\bar{x} = \Sigma fx / \Sigma f$ [For grouped data use mid-points of class intervals as x values]

Coding:

If $y = (x - a) / b$, then $\bar{y} = (\bar{x} - a) / b$, so $\bar{x} = a + b\bar{y}$. Coding simplifies calculations with large/awkward numbers. Note: coding DOES affect the mean and standard deviation.

11.3 Measures of Spread

Measure	Formula	Notes
Range	Maximum – Minimum	Simple but affected by outliers
Interquartile Range (IQR)	$Q_3 - Q_1$	Middle 50% of data; resistant to outliers
Variance (σ^2)	$\Sigma(x - \bar{x})^2 / n = \Sigma x^2/n - \bar{x}^2$	Mean of squared deviations from mean
Standard deviation (σ)	$\sigma = \sqrt{(\Sigma(x - \bar{x})^2/n)} = \sqrt{(\Sigma x^2/n - \bar{x}^2)}$	Most useful measure of spread; same units as data
Variance from freq table	$\Sigma f(x - \bar{x})^2 / \Sigma f = \Sigma fx^2 / \Sigma f - \bar{x}^2$	Use with frequency distribution

Standard deviation shortcut formula: $\sigma = \sqrt{(S_{xx}/n)}$ where $S_{xx} = \sum x^2 - (\sum x)^2/n$ [S_{xx} = sum of squared deviations — used in regression too]

Effect of coding $y = (x - a)/b$ on standard deviation: $\sigma_y = \sigma_x / b \rightarrow \sigma_x = b \times \sigma_y$. The standard deviation scales by $|b|$ only (not shifted by a).

11.4 Outliers

Outlier: A data point that is significantly different from the rest of the data. There is no single universal definition — context matters.

Definition	Outlier condition	Use
IQR method	$x < Q_1 - 1.5 \times \text{IQR}$ or $x > Q_3 + 1.5 \times \text{IQR}$	Most common in AS statistics
Standard deviation method	$x < \bar{x} - 2\sigma$ or $x > \bar{x} + 2\sigma$ (some use $\pm 3\sigma$)	When mean and SD known
Contextual	Values that are anomalous given subject knowledge	Always consider the context

Outliers may be: errors (should be removed); genuine extreme values (investigate carefully). Always discuss whether to include or exclude outliers and justify your decision.

11.5 Data Representations

Diagram	Data type	What it shows	Key features
Box plot (box-and-whisker)	Continuous/discrete	Median, Q_1 , Q_3 , range, outliers	Five-number summary; easy comparison of distributions
Histogram	Continuous grouped	Frequency density vs class width	Area $\propto$ frequency; unequal class widths common
Cumulative frequency graph	Continuous grouped	Running total of frequencies	Read off quartiles, percentiles
Frequency polygon	Discrete/continuous	Frequency vs midpoint of class	Useful for comparing two distributions
Stem-and-leaf diagram	Small discrete/continuous datasets	All raw data visible; ordered	Back-to-back for comparison
Scatter diagram	Bivariate data	Relationship between two variables	Look for correlation; identify outliers

Frequency density: **Frequency density = Frequency / Class width** [y-axis on a histogram — ensures area represents frequency]

Skewness:

Skew	Tail direction	Relationship of averages	Box plot: median position
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Positive (right) skew	Long tail to the RIGHT	Mode < Median < Mean	Median closer to Q_1 than Q_3
Negative (left) skew	Long tail to the LEFT	Mean < Median < Mode	Median closer to Q_3 than Q_1
Symmetric (no skew)	Both tails equal	Mean = Median = Mode	Median at centre of box

★ **Exam Tip:** When comparing two distributions, always comment on: (1) average (median or mean), (2) spread (IQR or range), (3) skew or shape, (4) outliers if relevant. Use numerical values from the data.

Topic 12: Probability

12.1 Basic Probability Definitions

Probability: A measure of the likelihood of an event, on a scale from 0 (impossible) to 1 (certain). $P(A) = \text{Number of favourable outcomes} / \text{Total number of equally likely outcomes}$ (classical definition).

Term	Definition	Formula
Complementary event	A' is the event 'A does not occur'	$P(A') = 1 - P(A)$
Union (A or B)	Event A or B (or both) occurs	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Intersection (A and B)	Both A and B occur	$P(A \cap B)$ — from Venn diagram or calculation
Mutually exclusive	A and B cannot both occur; $P(A \cap B) = 0$	$P(A \cup B) = P(A) + P(B)$
Exhaustive events	Events that cover all outcomes; $P(A_1 \cup A_2 \cup \dots \cup A_n) = 1$	Exhaustive set of mutually exclusive events sums to 1

Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ [Subtract the intersection to avoid double-counting]

12.2 Conditional Probability

Conditional probability: $P(A|B)$ = the probability of A given that B has occurred. The sample space is restricted to outcomes in B.

Conditional probability formula: $P(A|B) = P(A \cap B) / P(B)$ [Requires $P(B) > 0$]

Multiplication rule: $P(A \cap B) = P(A|B) \times P(B) = P(B|A) \times P(A)$ [Derived from the conditional probability formula]

Independent events: A and B are independent if the occurrence of one does not affect the other: $P(A|B) = P(A)$ and $P(B|A) = P(B)$. Equivalently: $P(A \cap B) = P(A) \times P(B)$.

Worked Example: Conditional probability

In a group of 100 students: 60 study Maths, 40 study Physics, 25 study both.

Find the probability a student studies Physics given they study Maths.

$$P(M) = 0.6, P(P) = 0.4, P(M \cap P) = 0.25$$

$$P(P|M) = P(P \cap M) / P(M) = 0.25/0.60 = 5/12 \approx 0.417$$

12.3 Venn Diagrams

Venn diagrams represent events as overlapping regions within a rectangle (sample space). The overlapping region shows the intersection $A \cap B$.

- Fill in the intersection $P(A \cap B)$ FIRST, then work outward.
- Numbers in each region represent probabilities or frequencies.
- Sum of all regions = 1 (probabilities) or n (frequencies).

Worked Example: Venn diagram problem

$P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.7$. Find $P(A \cap B)$, $P(A \text{ only})$, $P(\text{neither})$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.4 - 0.7 = 0.2$$

$$P(A \text{ only}) = P(A) - P(A \cap B) = 0.5 - 0.2 = 0.3$$

$$P(\text{neither}) = 1 - P(A \cup B) = 1 - 0.7 = 0.3$$

12.4 Tree Diagrams

Tree diagrams show sequences of events. Each branch shows a conditional probability. Rules:

- Probabilities on branches from the same node sum to 1.
- Multiply along a path to find the probability of that sequence of events.
- Add probabilities of different paths leading to the same outcome.

Worked Example: Tree diagram

Bag A has 3 red, 2 blue. Bag B has 1 red, 4 blue. A bag is chosen at random, then a ball.

$$P(\text{red}) = P(A) \times P(R|A) + P(B) \times P(R|B) = (1/2)(3/5) + (1/2)(1/5) = 3/10 + 1/10 = 4/10 = 2/5$$

$$P(\text{chose bag A} \mid \text{red ball}) = P(A \cap R) / P(R) = (3/10) / (4/10) = 3/4 \text{ (Bayes' theorem)}$$

★ **Exam Tip:** For 'without replacement' problems, update probabilities on second draw. e.g. if 1 red taken from 5 balls, remaining = 4 balls. For independence check: verify $P(A \cap B) = P(A) \times P(B)$ exactly.

Topic 13: Statistical Distributions

13.1 Discrete Random Variables

Random variable: A variable whose value is determined by the outcome of a random experiment. X (capital letter) denotes the variable; x (lowercase) denotes a specific value it takes.

Probability distribution: A table or formula that lists all possible values of X and their corresponding probabilities. All probabilities must sum to 1: $\sum P(X = x) = 1$.

Measure	Formula	Meaning
Mean / Expected value	$E(X) = \sum x P(X=x) = \sum xp$	The average value expected in the long run
Variance	$\text{Var}(X) = \sum x^2 P(X=x) - [E(X)]^2 = E(X^2) - [E(X)]^2$	Spread of the distribution
Standard deviation	$\sigma = \sqrt{\text{Var}(X)}$	Positive square root of variance

13.2 The Binomial Distribution

Binomial distribution: Models the number of successes in n independent trials, each with the same probability p of success. Written $X \sim B(n, p)$.

Conditions for binomial model (FIST):

- **F**ixed number of trials n .
- **I**ndependent trials — outcome of one trial doesn't affect another.
- **S**ame probability p of success on each trial.
- **T**wo outcomes only: success or failure.

Binomial P.D.F.: $P(X = r) = {}^nC_r \times p^r \times (1-p)^{n-r}$ [r = number of successes (0, 1, 2, ..., n); ${}^nC_r = n! / (r!(n-r)!)$]

Binomial mean and variance: $E(X) = np$ $\text{Var}(X) = np(1-p)$ $\sigma = \sqrt{np(1-p)}$ [Quick results — no need to sum xp]

Worked Example: Binomial probability

$X \sim B(10, 0.3)$. Find: (a) $P(X = 4)$, (b) $P(X \leq 3)$, (c) $P(X \geq 2)$.

(a) $P(X=4) = {}^{10}C_4 \times (0.3)^4 \times (0.7)^6 = 210 \times 0.0081 \times 0.117649 \approx 0.2001$

(b) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$ — use tables or calculator: ≈ 0.6496

(c) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - [P(X=0) + P(X=1)]$

$P(X=0) = (0.7)^{10} \approx 0.0282$; $P(X=1) = 10 \times (0.3) \times (0.7)^9 \approx 0.1211$

$P(X \geq 2) = 1 - 0.0282 - 0.1211 = 0.8507$

The **cumulative binomial distribution** $P(X \leq r)$ can be found using calculator tables (CDF function). For $P(X \geq r)$: use $P(X \geq r) = 1 - P(X \leq r-1)$. For $P(a \leq X \leq b)$: use $P(X \leq b) - P(X \leq a-1)$.

★ **Exam Tip:** Always state the distribution clearly: $X \sim B(n, p)$. Check all four FIST conditions before using the binomial model in a real-world problem.

Topic 14: Statistical Hypothesis Testing

14.1 The Hypothesis Testing Framework

Hypothesis test: A formal procedure for testing whether sample evidence is consistent with a specific claim about a population parameter.

Component	Definition	Example
Null hypothesis H_0	The statement to be tested — assumed true unless evidence shows otherwise	$H_0: p = 0.3$ (probability of success is 0.3)
Alternative hypothesis H_1	What we conclude if we reject H_0 — specifies the direction of the test	$H_1: p > 0.3$ (one-tailed) or $H_1: p \neq 0.3$ (two-tailed)
Test statistic	The observed value from the sample used to test H_0	X = number of successes in the sample
Significance level (α)	The threshold probability for rejecting H_0 — typically 5% or 1%	$\alpha = 0.05$ (5% significance level)
p-value	The probability of observing a result as extreme as the test statistic, assuming H_0 is true	If $p\text{-value} < \alpha$, reject H_0
Critical region	The set of values of the test statistic that lead to rejection of H_0	Reject H_0 if $X \leq 2$ or $X \geq 10$

14.2 One-Tailed and Two-Tailed Tests

Test type	H_1	Critical region location	When to use
One-tailed (right)	$p > p_0$	Upper tail of distribution	Testing for an increase
One-tailed (left)	$p < p_0$	Lower tail of distribution	Testing for a decrease
Two-tailed	$p \neq p_0$	Both tails; split α between them	Testing for any change

For a two-tailed test at 5% significance: use 2.5% in each tail. The critical region is split: values in the lower 2.5% AND the upper 2.5%.

14.3 Hypothesis Test for Binomial p

We test the probability of success p in a binomial distribution using observed data.

Step-by-step procedure:

- **Step 1 — Define the variable:** state $X \sim B(n, p)$ and what X represents.
- **Step 2 — State hypotheses:** write $H_0: p = p_0$ and $H_1: p (> \text{ or } < \text{ or } \neq) p_0$.
- **Step 3 — State significance level:** $\alpha = 0.05$ (5%) unless told otherwise.
- **Step 4 — Calculate probability:** Under H_0 , find $P(X \leq \text{observed value})$ or $P(X \geq \text{observed value})$.

- **Step 5 — Compare with α :** if $p\text{-value} \leq \alpha$, reject H_0 . Otherwise, do not reject H_0 .
- **Step 6 — Conclude in context:** state conclusion in words relating to the original problem.

Worked Example: Hypothesis test — one-tailed binomial

A coin is suspected of being biased toward heads. It is tossed 20 times and lands heads 14 times. Test at 5% significance level whether the coin is biased toward heads.

Let X = number of heads. $H_0: p = 0.5$, $H_1: p > 0.5$ (one-tailed, right)

Significance level: $\alpha = 0.05$

Under H_0 : $X \sim B(20, 0.5)$

$P(X \geq 14) = 1 - P(X \leq 13)$

From binomial tables or calculator: $P(X \leq 13) = 0.9423$

$P(X \geq 14) = 1 - 0.9423 = 0.0577$

$0.0577 > 0.05$, so do NOT reject H_0 .

Conclusion: There is insufficient evidence at the 5% significance level to conclude that the coin is biased toward heads.

Worked Example: Hypothesis test — using critical region

In the same test above: find the critical region at 5% level.

Under H_0 : $X \sim B(20, 0.5)$. Need $P(X \geq c) \leq 0.05$.

$P(X \geq 14) = 0.0577 > 0.05 \rightarrow 14$ NOT in critical region.

$P(X \geq 15) = 1 - P(X \leq 14) = 1 - 0.9793 = 0.0207 < 0.05 \rightarrow 15$ IS in critical region.

Critical region: $X \geq 15$. Since $14 < 15$, 14 is not in the critical region — do not reject H_0 .

The **actual significance level** is the probability of the critical region under H_0 . Due to the discrete nature of the binomial, the actual significance level is often less than the nominal level.

★ **Exam Tip:** Always write conclusions in the context of the question — never just say 'reject H_0 '. The conclusion must include: 'there is/is not sufficient evidence at the X% level to suggest that p (increased/decreased/changed)...'. Avoid saying the null hypothesis is 'proved' or 'disproved' — we only 'reject' or 'fail to reject'.

Section 3

Mechanics

Edexcel 8MA0/02 · Topics 15–18: Kinematics · Forces · Newton's Laws · Variable Acceleration

Topic 15: Quantities, Units and Modelling

15.1 SI Units and Quantities

Quantity	Symbol	SI Unit	Unit symbol
Displacement	s	Metre	m
Distance	d	Metre	m
Time	t	Second	s
Velocity	v	Metres per second	m s^{-1}
Speed	v	Metres per second	m s^{-1}
Acceleration	a	Metres per second squared	m s^{-2}
Mass	m	Kilogram	kg
Force	F	Newton	$\text{N} = \text{kg m s}^{-2}$
Weight	W	Newton	N

Weight and mass: $W = mg$ [$g = 9.8 \text{ m s}^{-2}$ (use 9.8 unless told otherwise; 9.81 or 10 sometimes specified)]

Scalars and Vectors:

Scalar (magnitude only)	Vector (magnitude and direction)
Speed, distance, mass, time	Velocity, displacement, acceleration, force, momentum
Temperature, energy, power	Weight (force due to gravity), normal reaction

15.2 Mathematical Modelling

Mathematical models simplify real situations using assumptions. Common assumptions in AS Mechanics:

Modelling term	Assumption	Consequence
Particle	Object has negligible size — treated as a point mass	Can ignore rotational effects; all forces act at one point
Smooth surface	No friction between surfaces	Normal reaction is the only contact force
Rough surface	Friction acts opposing motion	Friction force $F \leq \mu R$ (μ = coefficient of friction)
Light string/rod	String or rod has negligible mass	Tension same throughout; weight not considered
Inextensible string	String cannot stretch	Particles connected by it have same speed and acceleration
Bead on wire	Bead can move freely along wire	Constrained to follow the path of the wire
Uniform	Even distribution of mass	Centre of mass at geometric centre
Laminar flow / smooth fluid	No turbulence	Air resistance can be ignored (when specified)

★ **Exam Tip:** Always state your assumptions and their implications. If the problem says 'smooth', friction = 0. If 'light', ignore weight of string/rod.

Topic 16: Kinematics — Motion in a Straight Line

16.1 Displacement, Velocity and Acceleration

Displacement: The distance moved in a specified direction from a reference point. A vector. Symbol: s . Units: m.

Velocity: Rate of change of displacement with time. A vector. $v = ds/dt$. Speed = |velocity|. Units: $m\ s^{-1}$.

Acceleration: Rate of change of velocity with time. A vector. $a = dv/dt = d^2s/dt^2$. Units: $m\ s^{-2}$.

Average speed = total distance / total time. Note: average speed $\neq$ average velocity in general (displacement vs distance).

16.2 Velocity-Time Graphs

Feature of v-t graph	What it represents
Gradient at any point	Acceleration at that instant
Area under the graph (above t-axis)	Distance travelled (displacement)
Area below t-axis	Displacement in negative direction (subtract from above if total displacement needed)
Straight line	Constant acceleration (uniform motion)
Horizontal line	Constant velocity (zero acceleration)
Steeper line	Greater acceleration
Line crossing t-axis	Object changes direction at that point

16.3 SUVAT Equations for Constant Acceleration

The SUVAT equations apply when acceleration is **constant**. Variables:

- s = displacement (m)
- u = initial velocity ($m\ s^{-1}$)
- v = final velocity ($m\ s^{-1}$)
- a = acceleration ($m\ s^{-2}$) — constant
- t = time (s)

Equation	Variables used	Missing variable
$v = u + at$	u, v, a, t	s
$s = ut + \frac{1}{2}at^2$	s, u, a, t	v
$s = vt - \frac{1}{2}at^2$	s, v, a, t	u
$v^2 = u^2 + 2as$	u, v, a, s	t

$s = \frac{1}{2}(u + v)t$	s, u, v, t	a
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Worked Example: SUVAT problem

A car accelerates from rest at 2 m s^{-2} for 8 seconds, then brakes uniformly to rest in 4 s. Find the total distance and maximum speed.

Phase 1: $u = 0, a = 2, t = 8$

$v = u + at = 0 + 2(8) = 16 \text{ m s}^{-1}$ (max speed)

$s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(2)(64) = 64 \text{ m}$

Phase 2: $u = 16, v = 0, t = 4$

$s_2 = \frac{1}{2}(u + v)t = \frac{1}{2}(16 + 0)(4) = 32 \text{ m}$

Total distance = $64 + 32 = \mathbf{96 \text{ m}}$; Maximum speed = $\mathbf{16 \text{ m s}^{-1}}$

★ **Exam Tip:** Choose the SUVAT equation that contains the three knowns and the one unknown. Always define a positive direction and be consistent. For free fall: $a = g = 9.8 \text{ m s}^{-2}$ downward.

16.4 Vertical Motion Under Gravity

For objects in free fall (air resistance ignored), $a = g$ downward. Choosing upward as positive: $a = -9.8 \text{ m s}^{-2}$.

Worked Example: Projectile thrown vertically

A ball is thrown vertically upward with speed 20 m s^{-1} from ground level ($g = 9.8 \text{ m s}^{-2}$).

Taking upward as positive: $u = 20, a = -9.8$

(a) Maximum height: at max height $v = 0$.

$v^2 = u^2 + 2as \rightarrow 0 = 400 - 19.6s \rightarrow s = 400/19.6 \approx 20.4 \text{ m}$

(b) Time to reach max height: $v = u + at \rightarrow 0 = 20 - 9.8t \rightarrow t = 20/9.8 \approx 2.04 \text{ s}$

(c) Time to return to ground: $s = 0$ again

$0 = 20t - 4.9t^2 \rightarrow t(20 - 4.9t) = 0 \rightarrow t = 0$ or $t = 20/4.9 \approx 4.08 \text{ s}$

16.5 Projectile Motion (2D)

Projectile: An object moving freely under gravity after being projected. Horizontal velocity is constant (no air resistance). Vertical motion has constant downward acceleration g .

Horizontal component: $x = u \cos \theta \times t$ (constant velocity) [Horizontal acceleration = 0 (no air resistance)]

Vertical component: $y = u \sin \theta \times t - \frac{1}{2}gt^2$ (SUVAT with $a = -g$) [$v_y = u \sin \theta - gt$; at max height $v_y = 0$]

Worked Example: Projectile problem

A ball is kicked at 15 m s^{-1} at 40° above the horizontal. Find max height and range ($g = 9.8 \text{ m s}^{-2}$).

Initial components: $u_x = 15 \cos 40^\circ \approx 11.49 \text{ m s}^{-1}$; $u_y = 15 \sin 40^\circ \approx 9.64 \text{ m s}^{-1}$

Max height: when $v_y = 0$: $t = u_y/g = 9.64/9.8 \approx 0.984 \text{ s}$

$y_{\text{max}} = u_y t - \frac{1}{2}gt^2 = 9.64(0.984) - \frac{1}{2}(9.8)(0.984)^2 \approx 4.74 \text{ m}$

Total time of flight: $y = 0 \rightarrow 0 = u_y t - \frac{1}{2}gt^2 \rightarrow t = 2u_y/g = 2(9.64)/9.8 \approx 1.967 \text{ s}$

Range: $x = u_x \times t = 11.49 \times 1.967 \approx 22.6 \text{ m}$

★ **Exam Tip:** Resolve projectile motion into independent horizontal and vertical components. They share the same time variable. At max height, vertical velocity = 0 (horizontal velocity unchanged).

Topic 17: Forces and Newton's Laws of Motion

17.1 Types of Forces

Force	Description	Direction
Weight ($W = mg$)	Gravitational force on a mass m	Always vertically downward
Normal reaction (N or R)	Contact force perpendicular to surface	Perpendicular to the surface
Friction (F or f)	Resisting force opposing motion or tendency to move	Parallel to surface, opposing motion/tendency
Tension (T)	Force in a stretched string/rope	Along the string, toward the attachment point
Thrust/compression	Force in a compressed strut/rod	Along the rod, away from attachment
Driving force (D)	Engine force propelling a vehicle	Forward along direction of motion
Air resistance / drag	Resistance to motion through a fluid	Opposing direction of motion

17.2 Newton's Three Laws of Motion

Newton's First Law: An object remains at rest or moves with constant velocity unless acted upon by a resultant force. [Equilibrium condition: $\Sigma F = 0$]

Newton's Second Law: $F = ma$ (resultant force = mass $\times$ acceleration) [Resultant force in Newtons; mass in kg; acceleration in $m\ s^{-2}$]

Newton's Third Law: For every action, there is an equal and opposite reaction. [Forces act on DIFFERENT objects; they are equal in magnitude but opposite in direction]

17.3 Resolving Forces and Equilibrium

To analyse forces, resolve them into perpendicular components (usually horizontal and vertical, or parallel and perpendicular to a slope).

- Component of force F at angle θ to the horizontal: horizontal = $F \cos \theta$, vertical = $F \sin \theta$.
- For equilibrium: sum of forces in each direction = 0.
- For motion: apply $F = ma$ in the direction of motion.

Worked Example: Object on a slope

A box of mass 5 kg lies on a smooth slope inclined at 30° to the horizontal. A force P acts up the slope. Find P if the box is in equilibrium.

Forces: Weight $W = 5g = 49$ N (downward)

Normal reaction N (perpendicular to slope)

P (up the slope)

Resolving along the slope (positive up slope):

$$P - 5g \sin 30^\circ = 0 \rightarrow P = 5(9.8)(0.5) = 24.5 \text{ N}$$

Resolving perpendicular to slope:

$$N - 5g \cos 30^\circ = 0 \rightarrow N = 5(9.8)(\sqrt{3}/2) \approx 42.4 \text{ N}$$

17.4 Friction

Friction: A force that opposes motion (or potential motion) between two surfaces in contact. Only occurs on rough surfaces.

Friction inequality: $0 \leq F \leq \mu R$ [F = friction force; μ = coefficient of friction; R = normal reaction]

Limiting friction: $F_{\text{max}} = \mu R$ (when object is on the verge of moving or moving) [Limiting equilibrium: object about to move; moving object: friction = μR]

Worked Example: Friction on a horizontal surface

A crate of mass 8 kg is pulled along a rough horizontal floor ($\mu = 0.3$) by a horizontal force P . Find P for the crate to move at constant velocity.

At constant velocity: acceleration = 0, so resultant force = 0

Normal reaction: $N = 8g = 78.4$ N

Friction: $F = \mu N = 0.3 \times 78.4 = 23.52$ N

Resolving horizontally: $P - F = 0 \rightarrow P = 23.52 \text{ N} \approx 23.5 \text{ N}$

Worked Example: Friction on a slope

A particle of mass 2 kg sits on a rough slope ($\mu = 0.4$) at 35° to horizontal. Determine if the particle slides ($g = 9.8 \text{ m s}^{-2}$).

$$N = 2g \cos 35^\circ = 19.6 \times 0.819 = 16.05 \text{ N}$$

$$\text{Max static friction: } F_{\text{max}} = \mu N = 0.4 \times 16.05 = 6.42 \text{ N}$$

$$\text{Component of weight down slope: } 2g \sin 35^\circ = 19.6 \times 0.574 = 11.25 \text{ N}$$

Since $11.25 \text{ N} > 6.42 \text{ N}$, the particle SLIDES down the slope.

$$\text{Acceleration: } 2a = 2g \sin 35^\circ - \mu \times 2g \cos 35^\circ = 11.25 - 6.42 = 4.83$$

$$a = 4.83/2 = 2.42 \text{ m s}^{-2} \text{ down the slope}$$

17.5 Connected Particles

Two or more particles connected by a string (usually over a pulley). If the string is inextensible and the pulley is smooth, both particles have the same acceleration.

- Apply $F = ma$ to EACH particle separately (choose consistent positive direction for each).
- Tension T is the same throughout a light inextensible string over a smooth pulley.
- Add the equations to eliminate T ; solve for a ; substitute back to find T .

Worked Example: Atwood's machine (two hanging masses)

Mass $m_1 = 4 \text{ kg}$ and $m_2 = 6 \text{ kg}$ connected by a string over a smooth pulley ($g = 9.8 \text{ m s}^{-2}$).

Find the acceleration and tension.

m_1 accelerates upward; m_2 accelerates downward. Let a = acceleration.

For m_1 (upward positive): $T - m_1g = m_1a \rightarrow T - 39.2 = 4a \dots(1)$

For m_2 (downward positive): $m_2g - T = m_2a \rightarrow 58.8 - T = 6a \dots(2)$

Add (1) and (2): $58.8 - 39.2 = 10a \rightarrow 19.6 = 10a \rightarrow a = 1.96 \text{ m s}^{-2}$

From (1): $T = 39.2 + 4(1.96) = 39.2 + 7.84 = 47.04 \text{ N} \approx 47.0 \text{ N}$

Worked Example: Particle on surface connected to hanging mass

A 3 kg particle sits on a smooth horizontal table. A string over a pulley at the edge connects it to a 2 kg hanging mass. Find a and T .

For 3 kg (horizontal, tension pulls right): $T = 3a \dots(1)$

For 2 kg (vertical, weight pulls down): $2g - T = 2a \rightarrow 19.6 - T = 2a \dots(2)$

Add: $19.6 = 5a \rightarrow a = 3.92 \text{ m s}^{-2}$; $T = 3(3.92) = 11.76 \text{ N}$

★ **Exam Tip:** For connected particle problems: always draw separate force diagrams for each particle. Label ALL forces. State the direction you're taking as positive for each particle.

Topic 18: Variable Acceleration

18.1 Calculus and Kinematics

When acceleration is NOT constant, SUVAT equations cannot be used. Instead, use differentiation and integration.

Calculus relationships: $v = ds/dt$ $a = dv/dt = d^2s/dt^2$ [s = displacement, v = velocity, a = acceleration, t = time]

Integration relationships: $s = \int v \, dt + C_1$ $v = \int a \, dt + C_2$ [Use initial conditions to find constants of integration]

Summary of calculus in kinematics:

To find...	From...	Method
Velocity v	Displacement s as function of t	Differentiate s with respect to t : $v = ds/dt$
Acceleration a	Velocity v as function of t	Differentiate v with respect to t : $a = dv/dt$
Acceleration a	Displacement s as function of t	Differentiate twice: $a = d^2s/dt^2$
Velocity v	Acceleration a as function of t	Integrate a with respect to t : $v = \int a \, dt + C$
Displacement s	Velocity v as function of t	Integrate v with respect to t : $s = \int v \, dt + C$
Displacement s	Acceleration a as function of t	Integrate twice: $s = \int \int a \, dt \, dt + Ct + D$

18.2 Finding Velocity and Displacement

Worked Example: Differentiating to find velocity and acceleration

A particle moves along a straight line. Its displacement is $s = t^3 - 6t^2 + 9t + 2$ (m) at time t (s).

Velocity: $v = ds/dt = 3t^2 - 12t + 9$

Acceleration: $a = dv/dt = 6t - 12$

When is the particle at rest? $v = 0$: $3t^2 - 12t + 9 = 0 \rightarrow 3(t-1)(t-3) = 0 \rightarrow t = 1$ or $t = 3$

Direction of motion: $v > 0$ for $t < 1$ (positive), $v < 0$ for $1 < t < 3$ (negative), $v > 0$ for $t > 3$

The particle CHANGES DIRECTION at $t = 1$ and $t = 3$.

Acceleration at $t = 2$: $a = 6(2) - 12 = 0$ (the particle has zero acceleration at $t = 2$)

Worked Example: Integrating to find displacement

A particle starts from rest at the origin. Its acceleration is $a = 4t - 6 \text{ m s}^{-2}$ for $t \geq 0$.

Find v and s in terms of t .

$$v = \int (4t - 6) dt = 2t^2 - 6t + C$$

At $t = 0$, $v = 0$ (starts from rest): $0 = 0 - 0 + C \rightarrow C = 0$

$$\text{So } v = 2t^2 - 6t$$

$$s = \int (2t^2 - 6t) dt = \frac{2}{3}t^3 - 3t^2 + D$$

At $t = 0$, $s = 0$ (at origin): $0 = 0 - 0 + D \rightarrow D = 0$

$$\text{So } s = \frac{2}{3}t^3 - 3t^2$$

18.3 Maximum, Minimum Speed and Distance

The techniques of differentiation for optimisation apply directly to kinematics:

- **Maximum/minimum speed:** set $dv/dt = 0$ (i.e. $a = 0$) and solve for t ; check it's a max/min using the second derivative.
- **Maximum displacement:** set $v = ds/dt = 0$ and solve for t ; this is when the particle momentarily stops and changes direction.
- **Total distance (not displacement):** find all times when $v = 0$ (direction changes), calculate displacement in each phase separately, sum the absolute values.

Worked Example: Total distance travelled

Using $s = t^3 - 6t^2 + 9t + 2$ from the previous example, find total distance in $0 \leq t \leq 4$.

Direction changes at $t = 1$ and $t = 3$.

$$s(0) = 2; s(1) = 1 - 6 + 9 + 2 = 6; s(3) = 27 - 54 + 27 + 2 = 2; s(4) = 64 - 96 + 36 + 2 = 6$$

$$\text{Distance } 0 \rightarrow 1: |s(1) - s(0)| = |6 - 2| = 4 \text{ m}$$

$$\text{Distance } 1 \rightarrow 3: |s(3) - s(1)| = |2 - 6| = 4 \text{ m}$$

$$\text{Distance } 3 \rightarrow 4: |s(4) - s(3)| = |6 - 2| = 4 \text{ m}$$

$$\text{Total distance} = 4 + 4 + 4 = \mathbf{12 \text{ m}} \text{ (but displacement from 0 to 4} = 6 - 2 = 4 \text{ m)}$$

18.4 Using Integration with Known Velocity or Acceleration

Worked Example: Finding displacement using definite integration

A particle has velocity $v = 6t - t^2 \text{ m s}^{-1}$. Find the distance travelled from $t = 0$ to $t = 6$.

$$v = 0: 6t - t^2 = t(6 - t) = 0 \rightarrow t = 0 \text{ or } t = 6$$

$v \geq 0$ for $0 \leq t \leq 6$ (no direction change), so distance = |displacement|.

$$\int_0^6 (6t - t^2) dt = [3t^2 - t^3/3]_0^6 = (108 - 72) - 0 = 36 \text{ m}$$

★ **Exam Tip:** Always check for direction changes when finding total DISTANCE (not displacement). If v changes sign in the interval, split into sub-intervals. Distance is always positive; displacement can be negative.

AS Mathematics: Key Formulae Reference

The following formulae are given in the Edexcel AS Mathematics formula booklet. Those marked * must be learned (not in booklet).

Pure Mathematics

Formula	Expression
Quadratic formula *	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$
Discriminant *	$\Delta = b^2 - 4ac$
Laws of indices *	$a^m \times a^n = a^{m+n}$; $a^m/a^n = a^{m-n}$; $(a^m)^n = a^{mn}$; $a^0 = 1$; $a^{-n} = 1/a^n$
Binomial expansion	$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + b^n$ ($n \in \mathbb{Z}^+$)
Arithmetic series *	$u_n = a + (n-1)d$; $S_n = n/2(2a + (n-1)d) = n/2(a+u_n)$
Geometric series *	$u_n = ar^{n-1}$; $S_n = a(1-r^n)/(1-r)$; $S_\infty = a/(1-r)$ for $ r < 1$
Sine rule	$a/\sin A = b/\sin B = c/\sin C$
Cosine rule	$a^2 = b^2 + c^2 - 2bc \cos A$
Area of triangle	$\frac{1}{2} ab \sin C$
Arc length / sector	$l = r\theta$; $A = \frac{1}{2}r^2\theta$ (θ in radians)
Trig identities *	$\sin^2\theta + \cos^2\theta = 1$; $\tan \theta = \sin \theta / \cos \theta$
Differentiation: power rule *	$d/dx(x^n) = nx^{n-1}$; $d/dx(e^{kx}) = ke^{kx}$; $d/dx(\ln x) = 1/x$
Product rule	$d/dx(uv) = uv' + vu'$
Quotient rule	$d/dx(u/v) = (vu' - uv') / v^2$
Chain rule	$dy/dx = (dy/du)(du/dx)$
Integration: power rule *	$\int x^n dx = x^{n+1}/(n+1) + C$; $\int e^{kx} dx = e^{kx}/k + C$; $\int 1/x dx = \ln x + C$
Definite integral / area *	$A = \int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$
Log laws *	$\log_a(xy) = \log_a x + \log_a y$; $\log_a(x/y) = \log_a x - \log_a y$; $\log_a(x^n) = n \log_a x$

Statistics

Formula	Expression
Mean *	$\bar{x} = \Sigma x/n$ or $\Sigma fx/\Sigma f$
Variance *	$\sigma^2 = \Sigma(x - \bar{x})^2/n = \Sigma x^2/n - \bar{x}^2 = S_{xx}/n$
Std deviation *	$\sigma = \sqrt{S_{xx}/n}$ where $S_{xx} = \Sigma x^2 - (\Sigma x)^2/n$
Coding *	$\bar{y} = (\bar{x} - a)/b \rightarrow \bar{x} = a + b\bar{y}$; $\sigma_x = b \times \sigma_y$
$P(A')$	$1 - P(A)$
Addition rule	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Conditional probability	$P(A B) = P(A \cap B) / P(B)$
Independence *	$P(A \cap B) = P(A) \times P(B)$

Binomial P.D.F.	$P(X=r) = {}^nC_r p^r (1-p)^{n-r}$ for $X \sim B(n,p)$
Binomial mean/variance *	$E(X) = np$; $\text{Var}(X) = np(1-p)$
IQR outlier rule *	Outlier if $x < Q_1 - 1.5 \times \text{IQR}$ or $x > Q_3 + 1.5 \times \text{IQR}$

Mechanics

Formula	Expression
SUVAT equations	$v = u + at$; $s = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2as$; $s = \frac{1}{2}(u+v)t$
Weight *	$W = mg$ ($g \approx 9.8 \text{ m s}^{-2}$)
Newton's 2nd law *	$F = ma$ (resultant force = mass $\times$ acceleration)
Friction *	$F \leq \mu R$; at limiting friction: $F = \mu R$
Projectile (horiz.) *	$x = u \cos \theta \times t$ (constant velocity)
Projectile (vert.) *	$y = u \sin \theta \times t - \frac{1}{2}gt^2$; $v_y = u \sin \theta - gt$
Calculus kinematics *	$v = ds/dt$; $a = dv/dt = d^2s/dt^2$; $s = \int v \, dt$; $v = \int a \, dt$

Section 1 (A-Level)

Pure Mathematics — Advanced Topics

Topics 19–26 · Algebraic Methods · Functions · Advanced Trigonometry · Parametric Equations · Further Differentiation · Further Integration · Numerical Methods · 3D Vectors

Topic 19: Algebraic Methods

Algebraic Division

Algebraic division (also called polynomial long division) is used to divide one polynomial by another. It is closely linked to the factor and remainder theorems from AS level.

Remainder Theorem: If $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$

Factor Theorem: If $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$

To divide $f(x)$ by $(ax + b)$: rewrite as a quotient $Q(x)$ and remainder R , so that $f(x) = (ax + b)Q(x) + R$. The degree of $Q(x)$ is one less than $f(x)$.

Worked Example: Polynomial Division

Divide $f(x) = 2x^3 + 3x^2 - 11x - 6$ by $(x + 3)$.

Step 1: Set up long division. How many times does x go into $2x^3$? Answer: $2x^2$.

Step 2: Multiply $(x + 3)$ by $2x^2 = 2x^3 + 6x^2$. Subtract: $(2x^3 + 3x^2) - (2x^3 + 6x^2) = -3x^2$.

Step 3: Bring down $-11x$. Divide: x into $-3x^2 \rightarrow -3x$. Multiply: $-3x(x + 3) = -3x^2 - 9x$. Subtract: $-11x - (-9x) = -2x$.

Step 4: Bring down -6 . Divide: x into $-2x \rightarrow -2$. Multiply: $-2(x + 3) = -2x - 6$. Subtract: remainder = 0.

Result: $2x^3 + 3x^2 - 11x - 6 = (x + 3)(2x^2 - 3x - 2) = (x + 3)(2x + 1)(x - 2)$.

Partial Fractions

Partial fractions decompose a rational expression into simpler fractions. The degree of the numerator must be less than the degree of the denominator (if not, divide first).

Form of denominator	Partial fraction form
$(ax+b)(cx+d)$	$A/(ax+b) + B/(cx+d)$
$(ax+b)^2$	$A/(ax+b) + B/(ax+b)^2$
$(ax+b)(cx^2+d)$ [irreducible quadratic]	$A/(ax+b) + (Bx+C)/(cx^2+d)$

$$(ax+b)^3$$

$$A/(ax+b) + B/(ax+b)^2 + C/(ax+b)^3$$

Worked Example: Partial Fractions — Distinct Linear Factors

Express $(5x - 1) / [(x+1)(x-2)]$ as partial fractions.

Write $(5x-1)/[(x+1)(x-2)] = A/(x+1) + B/(x-2)$.

Multiply through: $5x - 1 = A(x-2) + B(x+1)$.

Let $x = 2$: $10 - 1 = 3B \rightarrow B = 3$.

Let $x = -1$: $-5 - 1 = -3A \rightarrow A = 2$.

Answer: $2/(x+1) + 3/(x-2)$.

Worked Example: Partial Fractions — Repeated Linear Factor

Express $(3x^2+2x+1) / [(x-1)^2(x+2)]$ as partial fractions.

Write as $A/(x-1) + B/(x-1)^2 + C/(x+2)$.

Multiply: $3x^2+2x+1 = A(x-1)(x+2) + B(x+2) + C(x-1)^2$.

Let $x=1$: $6 = 3B \rightarrow B = 2$.

Let $x=-2$: $12-4+1 = 9C \rightarrow C = 1$.

Expand and compare x^2 coefficients: $3 = A + C \rightarrow A = 2$.

Answer: $2/(x-1) + 2/(x-1)^2 + 1/(x+2)$.

★ **Exam Tip:** If the degree of the numerator $\geq$ degree of denominator, perform algebraic division first, then apply partial fractions to the remainder term.

Proof by Contradiction

Proof by contradiction: assume the negation of what you want to prove is true, then derive a logical contradiction. This proves the original statement must be true.

Worked Example: Proof by Contradiction: $\sqrt{2}$ is Irrational

Assume $\sqrt{2}$ is rational. Then $\sqrt{2} = p/q$ where p, q are integers with no common factors (in lowest terms).

Squaring: $2 = p^2/q^2$, so $p^2 = 2q^2$. Hence p^2 is even, so p is even.

Write $p = 2k$. Then $(2k)^2 = 2q^2 \rightarrow 4k^2 = 2q^2 \rightarrow q^2 = 2k^2$. Hence q^2 is even, so q is even.

But p and q are both even — contradicting our assumption that p/q is in lowest terms.

Contradiction. Therefore $\sqrt{2}$ is irrational. ■

Worked Example: Proof by Contradiction: Infinitely Many Primes

Assume there are finitely many primes: $p_1, p_2, \dots, p_n$.

Construct $N = p_1 \times p_2 \times \dots \times p_n + 1$.

N is not divisible by any p_i (remainder 1 each time), so N has no prime factor in our list.

Either N is prime (not in our list) or has a prime factor not in our list.

Either way, contradiction — our list was not complete. So there are infinitely many primes. ■

★ **Exam Tip:** In contradiction proofs: clearly state your assumption, derive the contradiction step-by-step, and end with 'This is a contradiction, therefore...'

Topic 20: Functions

Functions, Domain and Range

Function: A rule that maps each input (from the domain) to exactly one output (in the range).

Domain: The set of all possible input values of a function.

Range (Codomain): The set of all possible output values of a function.

One-to-one function: Each element of the domain maps to a unique element of the range (passes both horizontal and vertical line tests). Required for an inverse to exist.

Many-to-one function: Multiple domain values map to the same range value (e.g. $f(x) = x^2$, where $f(2) = f(-2) = 4$).

Composite Functions

Composite Function: $fg(x) = f(g(x))$ [apply g first, then f]

The domain of fg is all x in the domain of g such that $g(x)$ is in the domain of f . Note: $fg(x) \neq gf(x)$ in general — order matters.

Worked Example: Composite Functions

$f(x) = 2x + 1$ and $g(x) = x^2 - 3$ (both defined for $x \in \mathbb{R}$).

Find $fg(x)$: $fg(x) = f(g(x)) = f(x^2 - 3) = 2(x^2 - 3) + 1 = 2x^2 - 5$.

Find $gf(x)$: $gf(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 3 = 4x^2 + 4x + 1 - 3 = 4x^2 + 4x - 2$.

Find $ff(x)$: $ff(x) = f(f(x)) = f(2x + 1) = 2(2x + 1) + 1 = 4x + 3$.

Inverse Functions

Inverse Function: $f^{-1}(x)$ exists when f is one-to-one; $ff^{-1}(x) = f^{-1}f(x) = x$

The graph of $y = f^{-1}(x)$ is the reflection of $y = f(x)$ in the line $y = x$. The domain of f^{-1} is the range of f , and the range of f^{-1} is the domain of f .

To find $f^{-1}(x)$: write $y = f(x)$, rearrange to make x the subject, then swap x and y .

Worked Example: Finding an Inverse Function

$f(x) = (3x - 1)/(x + 2)$, $x \neq -2$. Find $f^{-1}(x)$.

Let $y = (3x - 1)/(x + 2)$. Multiply both sides by $(x + 2)$: $y(x + 2) = 3x - 1$.

Expand: $xy + 2y = 3x - 1$. Rearrange for x : $xy - 3x = -1 - 2y \rightarrow x(y - 3) = -(1 + 2y)$.

So $x = -(1 + 2y)/(y - 3) = (1 + 2y)/(3 - y)$.

Swap x and y : $f^{-1}(x) = (1 + 2x)/(3 - x)$, $x \neq 3$.

Check: domain of f^{-1} = range of f ; range of f^{-1} = domain of f .

The Modulus Function

|x|: $|x| = x$ if $x \geq 0$; $|x| = -x$ if $x < 0$ [always non-negative]

Key properties: $|ab| = |a||b|$; $|a/b| = |a|/|b|$; $|a+b| \leq |a|+|b|$ (triangle inequality)

Solving $|f(x)| = g(x)$: consider two cases — $f(x) = g(x)$ and $-f(x) = g(x)$. Check solutions satisfy the original equation.

Solving $|f(x)| < g(x)$: equivalent to $-g(x) < f(x) < g(x)$ (if $g(x) > 0$).

Worked Example: Modulus Equation and Inequality

(a) Solve $|2x - 3| = 5$.

Case 1: $2x - 3 = 5 \rightarrow x = 4$.

Case 2: $-(2x - 3) = 5 \rightarrow -2x + 3 = 5 \rightarrow x = -1$.

Solutions: $x = 4$ or $x = -1$.

(b) Solve $|3x + 1| < 8$.

Write as double inequality: $-8 < 3x + 1 < 8$.

Subtract 1: $-9 < 3x < 7$. Divide by 3: $-3 < x < 7/3$.

Graph Transformations

Key transformations involving the modulus function:

Function	Transformation	Description
$y = f(x)$	Reflect positive x-half in y-axis	Ignore $x < 0$ part; reflect $x > 0$ part
$y = f(x) $	Reflect parts below x-axis upwards	Any part where $f(x) < 0$ is reflected
$y = f(x) + a$	Translate $y = f(x) $ up by a	Vertical shift
$y = f(x + a) $	Translate left by a , then reflect	Horizontal shift then modulus
$y = af(x)$	Vertical stretch by factor a	Every y-value multiplied by a
$y = f(ax)$	Horizontal stretch by factor $1/a$	Every x-value divided by a

★ **Exam Tip:** For sketch questions involving $|f(x)|$: first sketch $y = f(x)$, then reflect any parts below the x-axis upward. Label key coordinates (especially x-intercepts and vertex).

Topic 21: Advanced Trigonometry

Reciprocal Trigonometric Functions

Function	Definition	Domain (deg)	Range
cosec x (csc x)	$1 / \sin x$	$x \neq 0^\circ, 180^\circ, 360^\circ, \dots$	$ \text{cosec } x \geq 1$
sec x	$1 / \cos x$	$x \neq 90^\circ, 270^\circ, \dots$	$ \sec x \geq 1$
cot x	$\cos x / \sin x = 1/\tan x$	$x \neq 0^\circ, 180^\circ, \dots$	$\cot x \in \mathbb{R}$

Pythagorean Identities

Identity 1: $\sin^2 x + \cos^2 x = 1$ [from AS level]

Identity 2: $1 + \tan^2 x = \sec^2 x$ [divide identity 1 by $\cos^2 x$]

Identity 3: $1 + \cot^2 x = \text{cosec}^2 x$ [divide identity 1 by $\sin^2 x$]

These identities are essential for simplifying expressions and solving equations. Common rearrangements: $\sec^2 x - 1 = \tan^2 x$; $\text{cosec}^2 x - 1 = \cot^2 x$.

Worked Example: Proving a Trigonometric Identity

Prove that $(\sec x - \cos x)(\text{cosec } x - \sin x) = \sin x \cos x / (1 + \tan^2 x)$.

LHS: $(1/\cos x - \cos x)(1/\sin x - \sin x)$.

$= ((1 - \cos^2 x)/\cos x) \times ((1 - \sin^2 x)/\sin x)$

$= (\sin^2 x / \cos x) \times (\cos^2 x / \sin x)$ [using $\sin^2 x = 1 - \cos^2 x$ etc.]

$= \sin x \cos x$.

RHS: $\sin x \cos x / \sec^2 x = \sin x \cos x \times \cos^2 x = \sin x \cos^3 x \dots$ re-examine.

Simpler: $(\sec x - \cos x) = \sin^2 x / \cos x$ and $(\text{cosec } x - \sin x) = \cos^2 x / \sin x$.

Product $= \sin^2 x \cos^2 x / (\sin x \cos x) = \sin x \cos x$. ✓

Addition Formulae

$\sin(A \pm B)$: $\sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B)$: $\cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B)$: $(\tan A \pm \tan B) / (1 \mp \tan A \tan B)$

Worked Example: Using Addition Formulae

Find the exact value of $\sin 75^\circ$.

$\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$.

$= (\sqrt{2}/2)(\sqrt{3}/2) + (\sqrt{2}/2)(1/2) = \sqrt{6}/4 + \sqrt{2}/4 = (\sqrt{6} + \sqrt{2})/4$.

Double Angle Formulae

$$\sin 2A: 2 \sin A \cos A$$

$$\cos 2A: \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A: 2 \tan A / (1 - \tan^2 A)$$

The three forms of $\cos 2A$ are all equivalent — choose whichever is most useful. Useful rearrangements: $\cos^2 A = (1 + \cos 2A)/2$; $\sin^2 A = (1 - \cos 2A)/2$.

Worked Example: Solving Equation Using Double Angle Formula

Solve $3\cos 2x + \cos x + 2 = 0$ for $0^\circ \leq x \leq 360^\circ$.

Use $\cos 2x = 2\cos^2 x - 1$: $3(2\cos^2 x - 1) + \cos x + 2 = 0$.

$6\cos^2 x - 3 + \cos x + 2 = 0 \rightarrow 6\cos^2 x + \cos x - 1 = 0$.

Factorise: $(3\cos x - 1)(2\cos x + 1) = 0$.

$\cos x = 1/3$: $x = 70.5^\circ$ or 289.5° .

$\cos x = -1/2$: $x = 120^\circ$ or 240° .

Solutions: $x = 70.5^\circ, 120^\circ, 240^\circ, 289.5^\circ$.

R sin(x + α) Form — Harmonic Form

$$a \sin x + b \cos x: R \sin(x + \alpha) \text{ [where } R = \sqrt{a^2 + b^2}, \tan \alpha = b/a, R > 0, 0 < \alpha < 90^\circ]$$

$$a \cos x + b \sin x: R \cos(x - \alpha) \text{ [where } R = \sqrt{a^2 + b^2}, \tan \alpha = b/a]$$

This form is used to find the maximum/minimum of trig expressions and to solve equations. Maximum value = R (when the angle inside = 90°); minimum = $-R$.

Worked Example: Writing in Harmonic Form and Solving

Write $3 \sin x + 4 \cos x$ in the form $R \sin(x + \alpha)$. Hence find the maximum value.

$R = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

$\tan \alpha = 4/3$, so $\alpha = \arctan(4/3) = 53.13^\circ$.

$3 \sin x + 4 \cos x = 5 \sin(x + 53.13^\circ)$.

Maximum value = 5, when $x + 53.13^\circ = 90^\circ$, i.e. $x = 36.87^\circ$.

Solve $3 \sin x + 4 \cos x = 3$ for $0^\circ \leq x \leq 360^\circ$:

$5 \sin(x + 53.13^\circ) = 3 \rightarrow \sin(x + 53.13^\circ) = 0.6$.

$x + 53.13^\circ = 36.87^\circ$ (discard, gives $x < 0$) or $180^\circ - 36.87^\circ = 143.13^\circ$.

$x = 143.13^\circ - 53.13^\circ = 90^\circ$. Also: $x + 53.13^\circ = 360^\circ + 36.87^\circ \rightarrow x = 343.7^\circ$.

Solutions: $x = 90^\circ$ and $x = 343.7^\circ$.

★ **Exam Tip:** Always verify $R > 0$ and α is in the correct quadrant. Draw the R triangle to avoid sign errors with α .

Topic 22: Parametric Equations

Introduction to Parametric Equations

Parametric equations: A curve is defined by $x = f(t)$ and $y = g(t)$, where t is a parameter. Each value of t gives a point (x, y) on the curve.

The parameter t often represents time, but can be any variable. Parametric equations can describe curves that cannot easily be written as $y = f(x)$.

Parametric equations	Cartesian equation	Notes
$x = r \cos t, y = r \sin t$ ($0 \leq t < 2\pi$)	$x^2 + y^2 = r^2$	Circle centre O, radius r
$x = a + r \cos t, y = b + r \sin t$	$(x-a)^2 + (y-b)^2 = r^2$	Circle centre (a, b)
$x = at^2, y = 2at$	$y^2 = 4ax$	Parabola
$x = a \sec t, y = b \tan t$	$x^2/a^2 - y^2/b^2 = 1$	Hyperbola
$x = t + 1, y = t^2 - 2$	$y = (x-1)^2 - 2$	Parabola ($t = x-1$)

Converting to Cartesian Form

Method: eliminate the parameter t between the two equations. Common techniques:

- **Direct substitution:** express t from one equation and substitute into the other.
- **Trigonometric identity:** use $\sin^2 t + \cos^2 t = 1$ or similar identities to eliminate t .

Worked Example: Converting Parametric to Cartesian

(a) $x = t + 2, y = t^2 - 1$. Find y in terms of x .

From x : $t = x - 2$. Substitute: $y = (x-2)^2 - 1 = x^2 - 4x + 3$.

(b) $x = 3 \cos t, y = 2 \sin t$. Find Cartesian equation.

Rearrange: $\cos t = x/3, \sin t = y/2$.

Use identity: $\cos^2 t + \sin^2 t = 1 \rightarrow (x/3)^2 + (y/2)^2 = 1$.

Cartesian equation: $x^2/9 + y^2/4 = 1$ (an ellipse).

Parametric Differentiation

$dy/dx: (dy/dt) \div (dx/dt)$ [chain rule: $dy/dx = (dy/dt)/(dx/dt)$]

$d^2y/dx^2: (d/dt)(dy/dx) \div (dx/dt)$ [differentiate dy/dx w.r.t. t , then divide by dx/dt]

Worked Example: Parametric Differentiation

$x = t^2 + 2t, y = t^3 - 3t$. Find dy/dx and the equation of the tangent at $t = 1$.

$dx/dt = 2t + 2, dy/dt = 3t^2 - 3$.

$dy/dx = (3t^2 - 3) / (2t + 2) = 3(t^2 - 1) / 2(t + 1) = 3(t-1)(t+1) / 2(t+1) = 3(t-1)/2$ [$t \neq -1$].

At $t = 1$: $dy/dx = 3(1-1)/2 = 0$ (horizontal tangent).

Point at $t=1$: $x = 1+2 = 3, y = 1-3 = -2$. Tangent: $y = -2$.

Worked Example: Finding Stationary Points Parametrically

$x = \cos t$, $y = \sin 2t$. Find stationary points for $0 \leq t \leq 2\pi$.

$dy/dx = (dy/dt)/(dx/dt) = 2 \cos 2t / (-\sin t)$.

Stationary points where $dy/dt = 0$: $2 \cos 2t = 0 \rightarrow \cos 2t = 0 \rightarrow 2t = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$.

$t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

Check $dx/dt = -\sin t \neq 0$ at these values (it doesn't).

At $t=\pi/4$: $x=\cos(\pi/4)=\sqrt{2}/2$, $y=\sin(\pi/2)=1$. Point: $(\sqrt{2}/2, 1)$.

At $t=3\pi/4$: $x=-\sqrt{2}/2$, $y=1$. At $t=5\pi/4$: $x=-\sqrt{2}/2$, $y=-1$. At $t=7\pi/4$: $x=\sqrt{2}/2$, $y=-1$.

★ **Exam Tip:** In parametric differentiation, remember to also check for vertical tangents (where $dx/dt = 0$ but $dy/dt \neq 0$) and cusps (both $= 0$).

Topic 23: Further Differentiation

Differentiating Trigonometric Functions

Function	Derivative	Notes
$\sin x$	$\cos x$	x in radians
$\cos x$	$-\sin x$	x in radians
$\tan x$	$\sec^2 x$	quotient rule on $\sin x/\cos x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$	
$\sec x$	$\sec x \tan x$	
$\cot x$	$-\operatorname{cosec}^2 x$	
$\sin(ax+b)$	$a \cos(ax+b)$	chain rule
$\cos(ax+b)$	$-a \sin(ax+b)$	chain rule
$\tan(ax+b)$	$a \sec^2(ax+b)$	chain rule

Implicit Differentiation

When a relationship between x and y cannot be written as $y = f(x)$ explicitly, differentiate both sides with respect to x using the chain rule: $d/dx[f(y)] = f'(y) \times dy/dx$.

Key rule: $d/dx[f(y)] = f'(y) \cdot dy/dx$ [chain rule applied to functions of y]

Worked Example: Implicit Differentiation

Find dy/dx for the curve $x^2 + y^2 + 3xy = 7$.

Differentiate each term w.r.t. x :

$d/dx(x^2) = 2x$.

$d/dx(y^2) = 2y \cdot dy/dx$.

$d/dx(3xy) = 3(y + x \cdot dy/dx)$ [product rule].

So: $2x + 2y(dy/dx) + 3y + 3x(dy/dx) = 0$.

Collect dy/dx terms: $dy/dx(2y + 3x) = -2x - 3y$.

$dy/dx = -(2x + 3y) / (2y + 3x)$.

Worked Example: Tangent to an Implicit Curve

Find the equation of the tangent to $x^2 + xy + y^2 = 7$ at $(1, 2)$.

Differentiate: $2x + (y + x \cdot dy/dx) + 2y \cdot dy/dx = 0$.

$dy/dx(x + 2y) = -2x - y \rightarrow dy/dx = -(2x+y)/(x+2y)$.

At $(1, 2)$: $dy/dx = -(2+2)/(1+4) = -4/5$.

Tangent: $y - 2 = -4/5(x - 1) \rightarrow 5y - 10 = -4x + 4 \rightarrow 4x + 5y = 14$.

Parametric Differentiation (Second Derivative)

$$d^2y/dx^2: d/dx(dy/dx) = [d/dt(dy/dx)] / (dx/dt)$$

Worked Example: Second Derivative Parametrically

$x = t^3$, $y = t^2 - 2$. Find d^2y/dx^2 .

$dx/dt = 3t^2$, $dy/dt = 2t$.

$dy/dx = 2t / 3t^2 = 2/(3t)$.

$d/dt(dy/dx) = d/dt(2/(3t)) = -2/(3t^2)$.

$d^2y/dx^2 = [-2/(3t^2)] / (3t^2) = -2/(9t^4)$.

Connected Rates of Change

Chain rule for rates: $dA/dt = dA/dr \times dr/dt$ [or any combination of variables]

Used when quantities change with time and you need to link their rates.

Worked Example: Connected Rates of Change

Water fills a spherical tank (radius r cm) at $50 \text{ cm}^3/\text{s}$. Find dr/dt when $r = 5$ cm.

Volume of sphere: $V = (4/3)\pi r^3$.

Differentiate: $dV/dr = 4\pi r^2$.

$dV/dt = 50$ (given).

By chain rule: $dV/dt = (dV/dr)(dr/dt) \rightarrow 50 = 4\pi r^2 \times dr/dt$.

At $r = 5$: $50 = 4\pi(25) \times dr/dt \rightarrow dr/dt = 50/(100\pi) = 1/(2\pi) \text{ cm/s} \approx 0.159 \text{ cm/s}$.

★ **Exam Tip:** Draw a diagram and write down the known formula. Differentiate to get a relationship between rates, then substitute the given values.

Topic 24: Further Integration

Integration of Trigonometric Functions

Integral	Result	Notes
$\int \sin x \, dx$	$-\cos x + C$	
$\int \cos x \, dx$	$\sin x + C$	
$\int \sin(ax+b) \, dx$	$-(1/a)\cos(ax+b) + C$	divide by a
$\int \cos(ax+b) \, dx$	$(1/a)\sin(ax+b) + C$	divide by a
$\int \sec^2 x \, dx$	$\tan x + C$	
$\int \operatorname{cosec} x \cot x \, dx$	$-\operatorname{cosec} x + C$	
$\int \sec x \tan x \, dx$	$\sec x + C$	
$\int \sin^2 x \, dx$	$x/2 - \sin(2x)/4 + C$	use $\cos 2x = 1 - 2\sin^2 x$
$\int \cos^2 x \, dx$	$x/2 + \sin(2x)/4 + C$	use $\cos 2x = 2\cos^2 x - 1$

Integration by Parts

Integration by Parts: $\int u \, dv = uv - \int v \, du$

Choose u using the LIATE priority order (Logarithm, Inverse trig, Algebraic, Trig, Exponential) — pick u from higher in the list. Let dv = the rest.

Worked Example: Integration by Parts

Find $\int x e^x \, dx$.

Let $u = x \rightarrow du = dx$. Let $dv = e^x \, dx \rightarrow v = e^x$.

$\int x e^x \, dx = x e^x - \int e^x \, dx = x e^x - e^x + C = e^x(x - 1) + C$.

Worked Example: Integration by Parts Twice

Find $\int x^2 \sin x \, dx$.

Let $u = x^2$, $dv = \sin x \, dx \rightarrow v = -\cos x$.

$= -x^2 \cos x + \int 2x \cos x \, dx$.

Now apply IBP again: $u = 2x$, $dv = \cos x \, dx \rightarrow v = \sin x$.

$= -x^2 \cos x + [2x \sin x - \int 2 \sin x \, dx]$.

$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$.

Worked Example: Integration by Parts — $\ln x$

Find $\int \ln x \, dx$.

Write as $\int \ln x \cdot 1 \, dx$. Let $u = \ln x$, $dv = dx \rightarrow v = x$.

$= x \ln x - \int x \cdot (1/x) \, dx = x \ln x - \int 1 \, dx = x \ln x - x + C$.

Integration Using Partial Fractions

Decompose the integrand into partial fractions (Topic 19), then integrate each term separately. Remember: $\int 1/(ax+b) \, dx = (1/a) \ln|ax+b| + C$.

Worked Example: Integration via Partial Fractions

Find $\int (5x - 1)/[(x+1)(x-2)] dx$.

From Topic 19: $(5x-1)/[(x+1)(x-2)] = 2/(x+1) + 3/(x-2)$.

$$\begin{aligned}\int [2/(x+1) + 3/(x-2)] dx &= 2 \ln|x+1| + 3 \ln|x-2| + C \\ &= \ln|(x+1)^2(x-2)^3| + C.\end{aligned}$$

Integration Using Trig Identities

Use identities to rewrite integrands into integrable forms:

- $\sin^2 x = (1 - \cos 2x)/2$ and $\cos^2 x = (1 + \cos 2x)/2$
- $\sin x \cos x = (1/2) \sin 2x$
- $\sin A \cos B = (1/2)[\sin(A+B) + \sin(A-B)]$ (product-to-sum)

Worked Example: Integrating Using Identities

Find $\int_0^{\pi/4} \cos^2 x dx$.

$$\cos^2 x = (1 + \cos 2x)/2.$$

$$\begin{aligned}\int_0^{\pi/4} (1 + \cos 2x)/2 dx &= [x/2 + \sin(2x)/4]_0^{\pi/4} \\ &= (\pi/8 + \sin(\pi/2)/4) - 0 = \pi/8 + 1/4.\end{aligned}$$

Differential Equations (Separable)

Separable differential equation: A DE that can be written as $f(y) dy/dx = g(x)$, then integrated: $\int f(y) dy = \int g(x) dx + C$.

Worked Example: Solving a Separable DE

Solve $dy/dx = xy$, given $y = 2$ when $x = 0$.

Separate: $(1/y) dy = x dx$.

Integrate both sides: $\ln|y| = x^2/2 + C$.

$|y| = e^C \cdot e^{(x^2/2)}$. Let $A = \pm e^C$: $y = A e^{(x^2/2)}$.

Apply $y = 2$ when $x = 0$: $2 = A e^0 = A$. So $y = 2e^{(x^2/2)}$.

Worked Example: Exponential Growth/Decay DE

A population P grows at rate $dP/dt = kP$. Find P in terms of t , given $P = P_0$ at $t = 0$.

Separate: $(1/P) dP = k dt$. Integrate: $\ln P = kt + C$.

$P = Ae^{(kt)}$. At $t = 0$: $P_0 = A$. So $P = P_0 e^{(kt)}$.

If given $P = P_1$ at $t = t_1$, find k : $k = (1/t_1) \ln(P_1/P_0)$.

Topic 25: Numerical Methods

Locating Roots — Sign Change Method

A root of $f(x) = 0$ lies in $[a, b]$ if $f(a)$ and $f(b)$ have opposite signs (sign change) AND f is continuous on $[a, b]$. We say there is a root in (a, b) .

Sign Change: If $f(a) < 0 < f(b)$, then $\exists$ root in (a, b) by IVT [for continuous f]

Worked Example: Locating a Root

Show that $f(x) = x^3 - 2x - 5$ has a root between $x = 2$ and $x = 3$.

$$f(2) = 8 - 4 - 5 = -1 < 0.$$

$$f(3) = 27 - 6 - 5 = 16 > 0.$$

Sign change with f continuous on $[2, 3] \rightarrow$ root exists in $(2, 3)$. ✓

★ **Exam Tip:** Show the actual values $f(a)$ and $f(b)$, state they have opposite signs, confirm f is continuous, then state the conclusion. All four parts needed for full marks.

Decimal Search and Interval Bisection

Having located a root in (a, b) , narrow the interval by evaluating f at the midpoint. If $f(\text{midpoint})$ has the same sign as $f(a)$, the root is in $(\text{midpoint}, b)$; otherwise $(a, \text{midpoint})$. Repeat to desired accuracy.

Fixed-Point Iteration

Rearrange $f(x) = 0$ to the form $x = g(x)$. Then iterate: $x_{n+1} = g(x_n)$ starting from an initial estimate x_0 . The iteration converges if $|g'(x)| < 1$ near the root.

Fixed-Point Iteration: $x_{n+1} = g(x_n)$ [converges if $|g'(\text{root})| < 1$]

Worked Example: Fixed-Point Iteration

Find a root of $x^3 - 3x + 1 = 0$ using the rearrangement $x = (x^3 + 1)/3$, starting at $x_0 = 0.5$.

$$x_1 = (0.125 + 1)/3 = 1.125/3 = 0.375.$$

$$x_2 = (0.375^3 + 1)/3 = (0.0527 + 1)/3 = 0.351.$$

$$x_3 = (0.351^3 + 1)/3 = (0.0432 + 1)/3 = 0.348.$$

$$x_4 = 0.347 \text{ (converging to root } \approx 0.347).$$

$$\text{Check: } f(0.347) = 0.347^3 - 3(0.347) + 1 \approx 0.042 - 1.041 + 1 \approx 0.001 \approx 0. \checkmark$$

Newton-Raphson Method

Newton-Raphson: $x_{n+1} = x_n - f(x_n)/f'(x_n)$

Newton-Raphson converges quickly (quadratically) when x_0 is close to the root. It can fail if $f'(x_n) = 0$, or if x_0 is near a turning point (tangent is nearly horizontal), or if the root is not isolated.

Worked Example: Newton-Raphson Iteration

Use Newton-Raphson to find a root of $f(x) = x^3 - 2x - 5 = 0$, starting at $x_0 = 2$.

$$f'(x) = 3x^2 - 2.$$

$$x_1 = x_0 - f(x_0)/f'(x_0) = 2 - f(2)/f'(2) = 2 - (-1)/(12-2) = 2 + 1/10 = 2.1.$$

$$x_2 = 2.1 - f(2.1)/f'(2.1) = 2.1 - (9.261-4.2-5)/(3(4.41)-2) = 2.1 - 0.061/11.23 = 2.0946.$$

$$x_3 = 2.0946 - f(2.0946)/f'(2.0946) \approx 2.0946 \text{ (converged).}$$

Root ≈ 2.0946 (to 4 d.p.).

★ **Exam Tip:** Always verify the answer by substituting back: $|f(\text{answer})|$ should be very small. Show at least 2-3 iterations in exam answers.

Topic 26: Vectors in 3D

3D Vectors — Notation and Operations

In 3D, vectors have components in three directions: i (x-axis), j (y-axis), k (z-axis). A vector $a = xi + yj + zk = (x, y, z)$ in column notation.

Operation	Formula	Example
Magnitude	$ a = \sqrt{x^2 + y^2 + z^2}$	$ (2, -1, 3) = \sqrt{14}$
Addition/Subtraction	$(a_1, a_2, a_3) \pm (b_1, b_2, b_3) = (a_1 \pm b_1, \dots)$	$(1, 2, 3) + (2, -1, 0) = (3, 1, 3)$
Scalar multiple	$\lambda(x, y, z) = (\lambda x, \lambda y, \lambda z)$	$2(1, 2, 3) = (2, 4, 6)$
Unit vector	$\hat{a} = a/ a $	$\hat{a} = (1/\sqrt{14})(2, -1, 3)$
Position vector	$\vec{OA} = a; \vec{AB} = b - a$	FROM A TO B = $b - a$

Section 2 (A-Level)

Statistics — Advanced Topics

Topics 27–29 · Normal Distribution · Regression & Correlation · Hypothesis Testing for Normal Distribution

Topic 27: Normal Distribution

The Normal Distribution

Normal Distribution: $X \sim N(\mu, \sigma^2)$ — a continuous distribution with bell-shaped curve, symmetric about the mean μ . Parameter σ^2 is the variance (σ is the standard deviation).

Property	Detail
Shape	Bell-shaped, symmetric about μ
Mean = Median = Mode	All equal to μ
Total area under curve	= 1 (since it's a probability distribution)
Spread determined by	Standard deviation σ — larger $\sigma \rightarrow$ wider curve
68-95-99.7 rule	~68% within 1σ ; ~95% within 2σ ; ~99.7% within 3σ of μ
$P(X = x)$	= 0 (continuous distribution — area = probability)

Standard Normal Distribution $Z \sim N(0, 1)$

Standardising: $Z = (X - \mu) / \sigma$ [transforms $X \sim N(\mu, \sigma^2)$ to $Z \sim N(0, 1)$]

The standard normal distribution has mean 0 and variance 1. Tables (or calculators) give $\Phi(z) = P(Z \leq z)$.

Symmetry properties:

- $P(Z > z) = 1 - P(Z \leq z) = 1 - \Phi(z)$
- $P(Z \leq -z) = P(Z \geq z) = 1 - \Phi(z)$ (by symmetry of $N(0, 1)$)
- $P(a \leq Z \leq b) = \Phi(b) - \Phi(a)$

Worked Example: Normal Distribution Probability

$X \sim N(50, 16)$. Find (a) $P(X < 54)$; (b) $P(X > 47)$; (c) $P(45 \leq X \leq 58)$.

(a) Standardise: $Z = (54 - 50)/\sqrt{16} = 4/4 = 1$.

$P(X < 54) = P(Z < 1) = \Phi(1) = 0.8413$.

(b) $Z = (47 - 50)/4 = -0.75$.

$P(X > 47) = P(Z > -0.75) = P(Z < 0.75) = \Phi(0.75) = 0.7734$.

(c) $Z_1 = (45 - 50)/4 = -1.25$; $Z_2 = (58 - 50)/4 = 2$.

$P(45 \leq X \leq 58) = \Phi(2) - \Phi(-1.25) = 0.9772 - (1 - 0.8944) = 0.9772 - 0.1056 = 0.8716$.

Inverse Normal — Finding x from a Probability

Inverse: finding x : $x = \mu + z \times \sigma$ [where z satisfies $\Phi(z) =$ given probability]

Worked Example: Inverse Normal Problem

$X \sim N(40, 25)$. Find the value k such that $P(X < k) = 0.9$.

Standardise: $P(Z < (k - 40)/5) = 0.9$.

From tables: $\Phi(1.282) \approx 0.9$, so $(k - 40)/5 = 1.282$.

$k = 40 + 5 \times 1.282 = 40 + 6.41 = 46.41$.

Worked Example: Finding μ and σ Simultaneously

$X \sim N(\mu, \sigma^2)$. $P(X < 10) = 0.2$ and $P(X < 14) = 0.7$. Find μ and σ .

Standardise: $(10 - \mu)/\sigma = z_1$ where $\Phi(z_1) = 0.2 \rightarrow z_1 = -0.842$.

$(14 - \mu)/\sigma = z_2$ where $\Phi(z_2) = 0.7 \rightarrow z_2 = 0.524$.

So: $10 - \mu = -0.842\sigma \dots(1)$ and $14 - \mu = 0.524\sigma \dots(2)$.

$(2) - (1)$: $4 = 1.366\sigma \rightarrow \sigma = 4/1.366 = 2.928$.

From (1): $\mu = 10 + 0.842(2.928) = 10 + 2.467 = 12.47$.

★ **Exam Tip:** Always draw a sketch of the normal curve, shading the required region. This helps identify whether to use z directly or $1 - \Phi(z)$.

Topic 28: Regression and Correlation

Scatter Diagrams and Types of Correlation

Correlation type	Scatter pattern	PMCC value (r)
Strong positive	Points close to upward line	r close to +1
Weak positive	Points roughly upward trend	$0 < r < 0.5$ (approx)
No correlation	Random scatter	r close to 0
Weak negative	Points roughly downward trend	$-0.5 < r < 0$ (approx)
Strong negative	Points close to downward line	r close to -1
Non-linear	Curved pattern	r unreliable — use other methods

Product Moment Correlation Coefficient (PMCC)

r measures the strength and direction of a linear relationship. $r = +1$ means perfect positive linear correlation; $r = -1$ means perfect negative; $r = 0$ means no linear correlation (there could still be a non-linear pattern).

Least Squares Regression Line

Regression line y on x: $\hat{y} = a + bx$

The regression line of y on x minimises the sum of squared vertical residuals (deviations). It is used to predict y from x .

Key points about regression:

- **Interpolation** (predicting within data range): reliable.
- **Extrapolation** (predicting outside data range): unreliable — the pattern may not continue.
- The regression line of x on y is different — use it only when predicting x from y .
- **Causation**: correlation does not imply causation — a third variable may be responsible.

★ **Exam Tip:** Always interpret the gradient in context: '...for each additional unit of x , y is expected to increase/decrease by [gradient] on average.'

Topic 29: Hypothesis Testing — Normal Distribution

Sampling Distribution of the Mean

Population: The entire group being studied, with mean μ and variance σ^2 .

Sample mean $\bar{X}$: The mean of a random sample of size n from this population.

Distribution of $\bar{X}$: $\bar{X} \sim N(\mu, \sigma^2/n)$ [if $X \sim N(\mu, \sigma^2)$ and sample size is n]

The standard deviation of $\bar{X}$ is $\sigma/\sqrt{n}$, called the **standard error of the mean**. As sample size increases, the distribution of $\bar{X}$ becomes narrower (more precise).

By the Central Limit Theorem (CLT): even if X is not normal, for large n ($n \geq 30$), $\bar{X}$ is approximately normal. For the exam, if X is normal, $\bar{X}$ is exactly normal.

Hypothesis Test for the Mean

We test whether a sample provides evidence that the population mean has changed. The test statistic is:

Test statistic: $Z = (\bar{X} - \mu_0) / (\sigma/\sqrt{n})$ [or use $\bar{X} \sim N(\mu_0, \sigma^2/n)$ directly]

Step	What to do
1. Define H_0 and H_1	$H_0: \mu = \mu_0$ (null). $H_1: \mu < \mu_0$ or $\mu > \mu_0$ or $\mu \neq \mu_0$
2. Set significance level	$\alpha = 0.05$ (5%) or 0.01 (1%) or 0.10 (10%)
3. Calculate test statistic	Find $z = (\bar{x} - \mu_0)/(\sigma/\sqrt{n})$ where $\bar{x}$ is the sample mean
4. Find critical value/p-value	Compare z to critical value, OR find $P(\bar{X} \leq \bar{x})$ or $P(\bar{X} \geq \bar{x})$
5. Make decision	If $ z > z_{\text{crit}}$ (or $p\text{-value} < \alpha$): reject H_0
6. Conclude in context	State conclusion in the context of the original problem

Critical Values for Common Significance Levels

Significance level α	One-tailed z_{crit}	Two-tailed z_{crit} (each tail)
10% (0.10)	1.282	1.645
5% (0.05)	1.645	1.960
2.5% (0.025)	1.960	2.241
1% (0.01)	2.326	2.576

Worked Example: Hypothesis Test — One-Tailed

A machine produces bolts with lengths $X \sim N(25, 4)$ mm. After servicing, a sample of 16 bolts has $\bar{x} = 24.2$. Test at 5% significance whether the mean length has decreased.

$H_0: \mu = 25$. $H_1: \mu < 25$ (one-tailed, lower).

Under H_0 : $\bar{X} \sim N(25, 4/16) = N(25, 0.25)$, so std error = $\sqrt{0.25} = 0.5$.

Test statistic: $z = (24.2 - 25)/0.5 = -0.8/0.5 = -1.6$.

Critical value for 5% one-tailed (lower): $z_{\text{crit}} = -1.645$.

Since $-1.6 > -1.645$ (z does not exceed critical region), do not reject H_0 .

Conclusion: Insufficient evidence at 5% significance level that the mean length has decreased.

Worked Example: Hypothesis Test — Two-Tailed

A random variable $X \sim N(100, 225)$. A sample of 9 gives $\bar{x} = 111$. Test at 5% whether μ has changed.

$H_0: \mu = 100$. $H_1: \mu \neq 100$ (two-tailed).

Under H_0 : $\bar{X} \sim N(100, 225/9) = N(100, 25)$, std error = 5.

Test statistic: $z = (111 - 100)/5 = 11/5 = 2.2$.

Critical value for 5% two-tailed: $z_{\text{crit}} = 1.960$.

Since $|2.2| = 2.2 > 1.960$, we reject H_0 .

Conclusion: Significant evidence at 5% level that μ has changed (increased).

Hypothesis Test for Correlation Coefficient

To test whether there is a linear correlation in the population, we test $H_0: \rho = 0$.

Test type	H_1
Positive correlation	$\rho > 0$
Negative correlation	$\rho < 0$
Any correlation	$\rho \neq 0$

The test uses the sample PMCC r . At significance level α with n data pairs, compare r against critical values from the PMCC table (provided in exam). If $|r| > \text{critical value}$, reject $H_0: \rho = 0$.

Worked Example: Testing for Correlation

$n = 8$ data pairs, calculated $r = 0.72$. Test at 5% for positive correlation.

$H_0: \rho = 0$. $H_1: \rho > 0$ (one-tailed).

From PMCC tables: critical value for $n=8$, 5% one-tailed ≈ 0.6215 .

Since $0.72 > 0.6215$, reject H_0 .

Conclusion: Significant evidence of positive linear correlation at 5% level.

★ **Exam Tip:** PMCC hypothesis tests are always about ρ (population correlation), NOT r (sample correlation). Always state H_0 as $\rho = 0$ and look up the critical value from the table.

Section 3 (A-Level)

Mechanics — Advanced Topics

Topic 30: Moments

Moments of a Force

Moment of a force: The turning effect of a force about a point. Moment = Force $\times$ perpendicular distance from the point to the line of action of the force.

Moment: $M = F \times d$ [F in newtons, d in metres $\rightarrow$ M in N·m]

Moments are either **clockwise** or **anticlockwise**. By convention, anticlockwise is usually taken as positive.

Concept	Detail
Perpendicular distance d	Must be measured perpendicularly from the pivot to the force's line of action
Force at angle θ	Use perpendicular component: $M = Fd \sin \theta$ (where θ is angle to the beam)
Resultant moment	Sum of clockwise moments minus sum of anticlockwise moments

Principle of Moments (Equilibrium)

Principle of Moments: For a body in equilibrium: sum of clockwise moments = sum of anticlockwise moments (about any point)

For a rigid body in equilibrium:

- The resultant force in any direction = 0.
- The resultant moment about any point = 0.
- Take moments about a point to eliminate an unknown force at that point.

Worked Example: Beam on Two Supports

A uniform beam AB, length 6 m, weight 200 N, rests on supports at A and C where AC = 4 m. A load of 50 N hangs at B. Find the reaction forces R_A and R_C .

Let R_A = reaction at A, R_C = reaction at C.

Take moments about A (eliminates R_A):

$$R_C \times 4 = 200 \times 3 + 50 \times 6 \text{ [beam weight acts at midpoint = 3 m from A]}$$

$$4 R_C = 600 + 300 = 900 \rightarrow R_C = 225 \text{ N.}$$

$$\text{Resolve vertically: } R_A + R_C = 200 + 50 = 250.$$

$$R_A = 250 - 225 = 25 \text{ N.}$$

$$\text{Check by taking moments about C: } 25 \times 4 = 200 \times 1 + 50 \times 2 \dots 100 = 200 - 100 = 100 \checkmark$$

Worked Example: Non-uniform Rod

A non-uniform rod AB, length 5 m, weight 60 N acts at distance d from A. Rests on supports at A and B. Given reaction at A = 36 N. Find d .

$R_A = 36$ N, $R_B = 60 - 36 = 24$ N.

Take moments about A: $R_B \times 5 = 60 \times d \rightarrow 24 \times 5 = 60d \rightarrow 120 = 60d \rightarrow d = 2$ m.

The centre of mass is 2 m from A.

★ **Exam Tip:** Always take moments about a point where an unknown acts — this eliminates that unknown. For uniform rods/beams, the weight acts at the midpoint.

A-Level Additional Formula Reference

Pure Mathematics — A-Level Additional

Formula	Topic	★ In booklet?
Partial fractions: $A/(ax+b) + B/(cx+d)$	19 — Algebraic Methods	No ★
$fg(x) = f(g(x))$; domain of fg	20 — Functions	No ★
$ x = x$ if $x \geq 0$, $-x$ if $x < 0$	20 — Functions	No ★
$a \sin x + b \cos x = R \sin(x+\alpha)$, $R=\sqrt{a^2+b^2}$	21 — Adv. Trig	No ★
$\sin(A\pm B)$, $\cos(A\pm B)$, $\tan(A\pm B)$ formulae	21 — Adv. Trig	In booklet
$\sin 2A = 2 \sin A \cos A$; $\cos 2A = \cos^2 A - \sin^2 A$	21 — Adv. Trig	In booklet
$dy/dx = (dy/dt)/(dx/dt)$; parametric	22/23 — Parametric/Further Diff	No ★
$d/dx[f(y)] = f'(y) \cdot dy/dx$ (implicit)	23 — Further Diff	No ★
$\int u \, dv = uv - \int v \, du$ (parts)	24 — Further Integration	In booklet
Newton-Raphson: $x_{n+1} = x_n - f(x_n)/f'(x_n)$	25 — Numerical Methods	In booklet

Statistics — A-Level Additional

Formula	Topic	★ In booklet?
$X \sim N(\mu, \sigma^2)$; $Z=(X-\mu)/\sigma \sim N(0,1)$	27 — Normal Dist	No ★
$\bar{X} \sim N(\mu, \sigma^2/n)$ (sampling distribution)	29 — Hypothesis Testing	No ★
Test statistic: $Z = (\bar{X}-\mu_0)/(\sigma/\sqrt{n})$	29 — Hypothesis Testing	No ★

Mechanics — A-Level Additional

Formula	Topic	★ In booklet?
Moment = $F \times d$ (perpendicular distance)	30 — Moments	No ★
Principle of Moments: Σ clockwise = Σ anticlockwise	30 — Moments	No ★

Exam Strategy and Common Mistakes

General A-Level Exam Technique

- **Show all working.** Method marks are available even if the final answer is wrong. A correct answer with no working may receive zero marks.
- **'Exact' answers:** leave in surd/log/ π /e form — never round to decimals unless asked.
- **'Show that' questions:** every algebraic step must be shown; the answer is given.
- **'Hence' questions:** you must use the result of the previous part.
- **Formula booklet:** know what IS and is NOT in it — practice using it efficiently.
- **Proof questions:** state clearly what you're assuming, what you've shown, and the conclusion.
- **Context questions:** always interpret mathematical conclusions in context (especially statistics).
- **Calculus:** include + C for indefinite integrals; use correct limits for definite integrals.

Common Mistakes to Avoid

Topic	Common mistake	Correct approach
Partial fractions	Not dividing first when $\text{degree}(\text{num}) \geq \text{degree}(\text{den})$	Always divide first if $\text{degree}(\text{num}) \geq \text{degree}(\text{den})$
Implicit diff	Forgetting $\times dy/dx$ when differentiating $f(y)$	$d/dx[y^2] = 2y \cdot dy/dx$ (chain rule on y)
Integration by parts	Wrong choice of u (picking e^x over polynomial)	LIATE: log $\rightarrow$ inverse $\rightarrow$ algebraic $\rightarrow$ trig $\rightarrow$ exponential
Normal distribution	Using wrong tail probability	Draw a sketch — shade the required region
Hypothesis testing	Forgetting to conclude in context	'Evidence suggests μ has changed...' not just 'reject H_0 '
Moments	Using slant distance instead of perpendicular	Moment = $F \times$ perpendicular distance
Parametric diff	Forgetting to divide by dx/dt for d^2y/dx^2	$d^2y/dx^2 = [d/dt(dy/dx)] \div (dx/dt)$
Vectors 3D	$\rightarrow AB = a - b$ instead of $b - a$	$\rightarrow AB = b - a$ (destination minus start)
Newton-Raphson	Not checking convergence/iterations	Show at least 2–3 iterations and verify final answer

Good luck with your Edexcel A-Level Mathematics examinations!