Algebra

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 **[1 mark]**

Simplify $5a + 3a - 2a$

1 All three terms are 'lots of a', so they are like terms and can be collected.

2 Work with the numbers in front: $5 + 3 - 2 = 6$

3 So the answer is $6a$

Answer: $6a$

2 **[2 marks]**

Work out the value of

(a) $3x + 4$ when $x = 5$

(b) $2x^2$ when $x = 3$

1 **(a)** Replace x with 5: $3 \times 5 + 4$

2 $3 \times 5 = 15$, then $15 + 4 = 19$

3 **(b)** $2x^2$ means 'square x first, then double' (BIDMAS — indices before multiplying).

4 $3^2 = 9$, then $2 \times 9 = 18$

Answer: (a) 19 (b) 18

3 **[2 marks]**

Expand

(a) $4(x + 3)$

(b) $3(2y - 5)$

1 **(a)** Multiply *everything* inside the bracket by 4.

2 $4 \times x = 4x$ and $4 \times 3 = 12$, giving $4x + 12$

3 **(b)** $3 \times 2y = 6y$ and $3 \times (-5) = -15$

4 Keep the minus sign: $6y - 15$

Answer: (a) $4x + 12$ (b) $6y - 15$

4 **[2 marks]**

Solve $5x - 7 = 23$

1 Undo the −7 first by adding 7 to both sides: $5x = 30$

2 Undo the ×5 by dividing both sides by 5: $x = 6$

3 Check: $5 \times 6 - 7 = 30 - 7 = 23$ ✓

Answer: $x = 6$

5 **[3 marks]**

Factorise fully

(a) $6x + 15$

(b) $x^2 + 7x$

(c) $8ab - 12a$

1 'Factorise fully' means take out the *highest* common factor.

2 **(a)** 6 and 15 are both multiples of 3, so $6x + 15 = 3(2x + 5)$

3 **(b)** Both terms contain x, so $x^2 + 7x = x(x + 7)$

4 **(c)** The numbers share 4 and both terms share a, so the common factor is $4a$.

5 $8ab \div 4a = 2b$ and $12a \div 4a = 3$, giving $4a(2b - 3)$

Answer: (a) $3(2x + 5)$ (b) $x(x + 7)$ (c) $4a(2b - 3)$

6 **[3 marks]**

Here are the first four terms of a sequence.

5 9 13 17

(a) Write down the next two terms.

(b) Find an expression for the nth term.

(c) Is 100 a term of this sequence? You must give a reason.

1 **(a)** The sequence goes up by 4 each time, so the next terms are $17 + 4 = 21$ and $21 + 4 = 25$

2 **(b)** A constant difference of 4 means the nth term starts with $4n$.

3 $4n$ gives 4, 8, 12, 16 — each is 1 less than the sequence, so the nth term is $4n + 1$

4 **(c)** Set $4n + 1 = 100$, so $4n = 99$ and $n = 24.75$

5 n must be a whole number (you cannot have term number 24.75), so 100 is not in the sequence.

Answer: (a) 21 and 25 (b) $4n + 1$ (c) No — $4n + 1 = 100$ gives $n = 24.75$, which is not a whole number

7 **[3 marks]**

Solve $4(x - 2) = 2x + 10$

1 Expand the bracket on the left: $4x - 8 = 2x + 10$

2 Get the x terms on one side — subtract $2x$ from both sides: $2x - 8 = 10$

3 Add 8 to both sides: $2x = 18$

4 Divide both sides by 2: $x = 9$

5 Check: $4(9 - 2) = 28$ and $2 \times 9 + 10 = 28$ ✓

Answer: $x = 9$

8 **[3 marks]**

(a) Solve the inequality $5x - 3 \leq 17$

(b) Write down the largest integer that satisfies your inequality.

1 **(a)** Treat it exactly like an equation. Add 3 to both sides: $5x \leq 20$

2 Divide both sides by 5. Because 5 is positive, the inequality sign does not flip: $x \leq 4$

3 **(b)** $x \leq 4$ includes 4 itself, so the largest integer is 4.

Answer: (a) $x \leq 4$ (b) 4

9 **[4 marks]**

Solve the simultaneous equations

$3x + y = 14$

$x + y = 6$

1 The y terms are identical (both $+y$), so **subtract** the second equation from the first.

2 $(3x - x) + (y - y) = 14 - 6$, which gives $2x = 8$

3 So $x = 4$

4 Substitute $x = 4$ into the simpler equation $x + y = 6$: $4 + y = 6$, so $y = 2$

5 Check in the other equation: $3 \times 4 + 2 = 14$ ✓

Answer: $x = 4$, $y = 2$

10 **[5 marks]**

A rectangle has length $(2x + 3)$ cm and width $(x - 1)$ cm.

The perimeter of the rectangle is 34 cm.

(a) Show that $6x + 4 = 34$

(b) Solve your equation to find x.

(c) Work out the area of the rectangle.

1 **(a)** Perimeter = 2 × (length + width) = $2[(2x + 3) + (x - 1)]$

2 Inside the brackets: $2x + x = 3x$ and $3 - 1 = 2$, giving $2(3x + 2)$

3 Expanding: $6x + 4$, and this equals the perimeter 34, so $6x + 4 = 34$ as required.

4 **(b)** $6x = 30$, so $x = 5$

5 **(c)** Length = $2 \times 5 + 3 = 13$ cm and width = $5 - 1 = 4$ cm.

6 Area = $13 \times 4 = 52$ cm^2 (check: perimeter = $2(13 + 4) = 34$ ✓)

Answer: (b) $x = 5$ (c) 52 cm^2

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1 **[2 marks]**

Expand and simplify $(x + 5)(x - 3)$

1 Multiply every term in the first bracket by every term in the second.

2 $x \times x = x^2$, $x \times (-3) = -3x$, $5 \times x = 5x$, $5 \times (-3) = -15$

3 Collect the middle terms: $-3x + 5x = 2x$

Answer: $x^2 + 2x - 15$

2 **[2 marks]**

Simplify

(a) $x^5 \times x^3$

(b) $(2y^3)^4$

1 **(a)** Multiplying powers of the same letter means *adding* the indices: $5 + 3 = 8$, so x^8

2 **(b)** Everything inside the bracket is raised to the power 4 — including the 2.

3 $2^4 = 16$ and $(y^3)^4 = y^{12}$ (multiply the indices).

> **Answer:** (a) x^8 (b) $16y^{12}$

3 **[3 marks]**

Factorise

(a) $x^2 - 49$

(b) $2x^2 + 7x + 3$

1 **(a)** This is the difference of two squares: $x^2 - 7^2$

2 So $x^2 - 49 = (x + 7)(x - 7)$

3 **(b)** Multiply the outer numbers: $2 \times 3 = 6$. Find two numbers multiplying to 6 and adding to 7 — that is 6 and 1.

4 Split the middle term: $2x^2 + 6x + x + 3$

5 Factorise in pairs: $2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$

> **Answer:** (a) $(x + 7)(x - 7)$ (b) $(2x + 1)(x + 3)$

4 **[3 marks]**

Solve the simultaneous equations

$3x + 2y = 19$

$5x - 2y = 5$

1 The y terms are $+2y$ and $-2y$, so **adding** the equations eliminates y.

2 $(3x + 5x) + (2y - 2y) = 19 + 5$, giving $8x = 24$

3 So $x = 3$

4 Substitute into $3x + 2y = 19$: $9 + 2y = 19$, so $2y = 10$ and $y = 5$

5 Check in the second equation: $5 \times 3 - 2 \times 5 = 15 - 10 = 5$ ✓

> **Answer:** $x = 3$, $y = 5$

5 **[3 marks]**

Make x the subject of the formula

$$y = \frac{3x + 2}{x - 1}$$

1 Multiply both sides by $(x - 1)$ to clear the fraction: $y(x - 1) = 3x + 2$

2 Expand the left-hand side: $xy - y = 3x + 2$

3 Collect every term containing x on one side: $xy - 3x = y + 2$

4 Factorise the left-hand side: $x(y - 3) = y + 2$

5 Divide by $(y - 3)$: $x = \frac{y + 2}{y - 3}$

> **Answer:** $x = \frac{y + 2}{y - 3}$

6 **[4 marks]**

Simplify fully

$$\frac{x^2 - 9}{x^2 + x - 12}$$

1 Factorise the top. It is the difference of two squares: $x^2 - 9 = (x + 3)(x - 3)$

2 Factorise the bottom. Two numbers multiplying to −12 and adding to +1 are +4 and −3.

3 So $x^2 + x - 12 = (x + 4)(x - 3)$

4 The fraction becomes $\frac{(x + 3)(x - 3)}{(x + 4)(x - 3)}$

5 Cancel the common factor $(x - 3)$ from top and bottom.

> **Answer:** $\frac{x + 3}{x + 4}$

7 **[4 marks]**

$f(x) = 3x - 1$ and $g(x) = x^2 + 2$

(a) Work out fg(2)

(b) Find gf(x), giving your answer in the form $ax^2 + bx + c$

(c) Find $f^{-1}(x)$

1 **(a)** fg(2) means 'do g first, then f'. $g(2) = 2^2 + 2 = 6$

2 Then $f(6) = 3 \times 6 - 1 = 17$

3 **(b)** gf(x) means 'do f first', so substitute $3x - 1$ into g: $(3x - 1)^2 + 2$

4 $(3x - 1)^2 = 9x^2 - 6x + 1$, so $gf(x) = 9x^2 - 6x + 3$

5 **(c)** Write $y = 3x - 1$ and rearrange: $y + 1 = 3x$, so $x = \frac{y + 1}{3}$

6 Swap the letter back: $f^{-1}(x) = \frac{x + 1}{3}$

> **Answer:** (a) 17 (b) $9x^2 - 6x + 3$ (c) $\frac{x + 1}{3}$

8 **[4 marks]**

Here are the first five terms of a quadratic sequence.

3   8   15   24   35

Find an expression for the nth term.

1 First differences: 5, 7, 9, 11. Second differences: 2, 2, 2 — constant, so the sequence is quadratic.

2 The n^2 coefficient is half the second difference: $2 \div 2 = 1$, so the sequence starts with n^2.

3 Write out n^2: 1, 4, 9, 16, 25

4 Subtract from the sequence: $3 - 1 = 2$, $8 - 4 = 4$, $15 - 9 = 6$, $24 - 16 = 8$, $35 - 25 = 10$

5 That leftover sequence 2, 4, 6, 8, 10 has nth term $2n$.

6 Check $n = 4$: $4^2 + 2 \times 4 = 16 + 8 = 24$ ✓

Answer: $n^2 + 2n$

9 **[4 marks]**

Solve the inequality

$$x^2 - 5x - 14 > 0$$

1 First find the critical values by solving $x^2 - 5x - 14 = 0$

2 Two numbers multiplying to −14 and adding to −5 are −7 and +2, so $(x - 7)(x + 2) = 0$

3 The critical values are $x = 7$ and $x = -2$

4 Sketch the curve: it is a positive (U-shaped) parabola crossing the x-axis at −2 and 7.

5 The curve is **above** the axis (> 0) outside the two roots.

Answer: $x < -2$ or $x > 7$

10 **[5 marks]**

Prove that the sum of the squares of any two consecutive odd numbers is always 2 more than a multiple of 8.

1 Use algebra so the proof covers *every* pair, not just examples.

2 Any odd number can be written as $2n + 1$, so the next odd number is $2n + 3$ (where n is an integer).

3 Square each: $(2n + 1)^2 = 4n^2 + 4n + 1$ and $(2n + 3)^2 = 4n^2 + 12n + 9$

4 Add them: $8n^2 + 16n + 10$

5 Take out a factor of 8 from as much as possible: $8(n^2 + 2n + 1) + 2$

6 Since $n^2 + 2n + 1$ is an integer, $8(n^2 + 2n + 1)$ is a multiple of 8, so the total is always 2 more than a multiple of 8.

Answer: $(2n + 1)^2 + (2n + 3)^2 = 8(n + 1)^2 + 2$, which is 2 more than a multiple of 8