Expanding and Factorising

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 **[1 mark]**

Expand $3(x + 2)$

1. Multiply both terms inside the bracket by 3.
2. $3 \times x = 3x$ and $3 \times 2 = 6$

Answer: $3x + 6$

2 **[2 marks]**

Expand

(a) $5(2x - 3)$

(b) $x(x + 7)$

1. **(a)** $5 \times 2x = 10x$ and $5 \times (-3) = -15$
2. **(b)** $x \times x = x^2$ and $x \times 7 = 7x$

Answer: (a) $10x - 15$ (b) $x^2 + 7x$

3 **[2 marks]**

Expand and simplify $2(x + 4) + 3(x - 1)$

1. Expand each bracket: $2x + 8$ and $3x - 3$
2. Collect the x terms: $2x + 3x = 5x$
3. Collect the numbers: $8 - 3 = 5$

Answer: $5x + 5$

4 **[2 marks]**

Factorise $6x + 9$

1. The highest common factor of 6 and 9 is 3.
2. $6x \div 3 = 2x$ and $9 \div 3 = 3$

Answer: $3(2x + 3)$

5 **[3 marks]**

Factorise fully

(a) $10x - 15$

(b) $x^2 - 6x$

(c) $12ab + 8b$

1 **(a)** The highest common factor is 5: $5(2x - 3)$

2 **(b)** Both terms contain x: $x(x - 6)$

3 **(c)** The numbers share 4 and both terms contain b, so take out $4b$.

4 $12ab \div 4b = 3a$ and $8b \div 4b = 2$

Answer: (a) $5(2x - 3)$ (b) $x(x - 6)$ (c) $4b(3a + 2)$

6 **[3 marks]**

Expand and simplify $(x + 3)(x + 4)$

1 Multiply every term in the first bracket by every term in the second.

2 $x \times x = x^2$, $x \times 4 = 4x$

3 $3 \times x = 3x$, $3 \times 4 = 12$

4 Collect the middle terms: $4x + 3x = 7x$

Answer: $x^2 + 7x + 12$

7 **[3 marks]**

Expand and simplify $4(2x - 1) - 3(x - 5)$

1 First bracket: $4 \times 2x = 8x$ and $4 \times (-1) = -4$

2 Second bracket: be careful, the -3 multiplies both terms.

3 $-3 \times x = -3x$ and $-3 \times (-5) = +15$

4 Collect: $8x - 3x = 5x$ and $-4 + 15 = 11$

Answer: $5x + 11$

8 **[3 marks]**

Factorise $x^2 + 8x + 15$

1 Find two numbers that multiply to 15 and add to 8.

2 Options for 15: 1×15 (sum 16) and 3×5 (sum 8) ✓

3 So the brackets are $(x + 3)(x + 5)$

4 Check by expanding: $x^2 + 5x + 3x + 15 = x^2 + 8x + 15$ ✓

Answer: $(x + 3)(x + 5)$

9 **[4 marks]**

Expand and simplify

(a) $(x - 2)(x + 6)$

(b) $(2x + 1)(x - 3)$

1 **(a)** $x^2 + 6x - 2x - 12$

2 Collect: $6x - 2x = 4x$, giving $x^2 + 4x - 12$

3 **(b)** $2x \times x = 2x^2$, $2x \times (-3) = -6x$

4 $1 \times x = x$, $1 \times (-3) = -3$

5 Collect: $-6x + x = -5x$

Answer: (a) $x^2 + 4x - 12$ (b) $2x^2 - 5x - 3$

10 **[4 marks]**

A rectangle has length $(x + 5)$ cm and width $(x + 2)$ cm.

(a) Show that the area of the rectangle is $x^2 + 7x + 10$ cm^2.

(b) A square of side x cm is cut out of the rectangle. Write down a simplified expression for the area that is left.

1 **(a)** Area = $(x + 5)(x + 2)$

2 Expanding: $x^2 + 2x + 5x + 10$

3 Collect the middle terms: $x^2 + 7x + 10$ as required.

4 **(b)** The square has area $x \times x = x^2$ cm^2

5 Remaining area = $(x^2 + 7x + 10) - x^2 = 7x + 10$

Answer: (b) $(7x + 10)$ cm^2

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1 **[2 marks]**

Expand and simplify $(x + 4)(x - 4)$

1 $x^2 - 4x + 4x - 16$

2 The two middle terms cancel out — this is the difference of two squares.

Answer: $x^2 - 16$

2 **[2 marks]**

Factorise $x^2 - 81$

1 There is no x term and $81 = 9^2$, so use the difference of two squares.

2 $a^2 - b^2 = (a + b)(a - b)$ with $a = x$ and $b = 9$

Answer: $(x + 9)(x - 9)$

3 **[3 marks]**

Expand and simplify $(2x - 3)^2$

1. A square means the bracket multiplied by itself: $(2x - 3)(2x - 3)$
2. $2x \times 2x = 4x^2$
3. The two cross terms: $2x \times (-3) = -6x$, twice, giving $-12x$
4. $(-3) \times (-3) = +9$

Answer: $4x^2 - 12x + 9$

4 **[3 marks]**

Factorise $3x^2 + 10x + 8$

1. Multiply the end numbers: $3 \times 8 = 24$. Find two numbers multiplying to 24 and adding to 10: 6 and 4.
2. Split the middle term: $3x^2 + 6x + 4x + 8$
3. Factorise in pairs: $3x(x + 2) + 4(x + 2)$
4. Both pairs share $(x + 2)$, so the answer is $(3x + 4)(x + 2)$

Answer: $(3x + 4)(x + 2)$

5 **[4 marks]**

Expand and simplify $(x + 2)(x - 1)(x + 3)$

1. Multiply the first two brackets: $(x + 2)(x - 1) = x^2 + x - 2$
2. Now multiply that by $(x + 3)$, term by term.
3. $x(x^2 + x - 2) = x^3 + x^2 - 2x$
4. $3(x^2 + x - 2) = 3x^2 + 3x - 6$
5. Add and collect: $x^3 + 4x^2 + x - 6$

Answer: $x^3 + 4x^2 + x - 6$

6 **[4 marks]**

Factorise fully

(a) $2x^2 - 18$

(b) $4x^2 - 20x + 24$

1. **(a)** Take out the common factor 2 first: $2(x^2 - 9)$
2. The bracket is a difference of two squares: $2(x + 3)(x - 3)$
3. **(b)** Take out 4: $4(x^2 - 5x + 6)$
4. Two numbers multiplying to 6 and adding to -5 are -2 and -3.
5. So the answer is $4(x - 2)(x - 3)$

Answer: (a) $2(x + 3)(x - 3)$ (b) $4(x - 2)(x - 3)$

7 **[4 marks]**

Factorise $6x^2 - x - 12$

1 Multiply the end numbers: $6 \times (-12) = -72$

2 Find two numbers multiplying to -72 and adding to -1: -9 and $+8$

3 Split the middle term: $6x^2 - 9x + 8x - 12$

4 Factorise in pairs: $3x(2x - 3) + 4(2x - 3)$

5 So the answer is $(3x + 4)(2x - 3)$

Answer: $(3x + 4)(2x - 3)$

8 **[4 marks]**

Simplify fully $\dfrac{2x^2 + 7x + 3}{x^2 - 9}$

1 Factorise the top: $2 \times 3 = 6$, and 6 and 1 multiply to 6 and add to 7.

2 $2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$

3 Factorise the bottom as a difference of two squares: $(x + 3)(x - 3)$

4 Cancel the common factor $(x + 3)$.

Answer: $\dfrac{2x + 1}{x - 3}$

9 **[4 marks]**

Prove that $(n + 3)^2 - (n + 1)^2$ is always a multiple of 4 for any integer n.

1 Expand each square separately.

2 $(n + 3)^2 = n^2 + 6n + 9$

3 $(n + 1)^2 = n^2 + 2n + 1$

4 Subtract: $(n^2 + 6n + 9) - (n^2 + 2n + 1) = 4n + 8$

5 Factorise: $4n + 8 = 4(n + 2)$

6 Since $n + 2$ is an integer, the result is $4 \times$ an integer, so it is always a multiple of 4.

Answer: $4(n + 2)$, which is a multiple of 4 for every integer n

10 **[5 marks]**

(a) Expand and simplify $(2n + 1)^2 - (2n - 1)^2$

(b) Hence explain why the difference between the squares of any two consecutive odd numbers is always a multiple of 8.

1 **(a)** $(2n + 1)^2 = 4n^2 + 4n + 1$

2 $(2n - 1)^2 = 4n^2 - 4n + 1$

3 Subtracting: $(4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$

4 **(b)** $2n - 1$ and $2n + 1$ are consecutive odd numbers for any integer n.

5 Part (a) shows their squares differ by $8n$.

6 Since n is an integer, $8n$ is always a multiple of 8.

Answer: (a) $8n$ (b) consecutive odd numbers can be written as $2n - 1$ and $2n + 1$, and their squares differ by $8n$