

Solving Equations

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1

[1 mark]

Solve $x + 7 = 12$

1 Undo the +7 by subtracting 7 from both sides.

2 $x = 12 - 7 = 5$

Answer: $x = 5$

2

[2 marks]

Solve

(a) $3x = 21$

(b) $\frac{x}{4} = 5$

1 (a) Undo the $\times 3$ by dividing both sides by 3: $x = 7$

2 (b) Undo the $\div 4$ by multiplying both sides by 4: $x = 20$

Answer: (a) $x = 7$ (b) $x = 20$

3

[2 marks]

Solve $2x + 5 = 17$

1 Deal with the +5 first: subtract 5 from both sides to get $2x = 12$

2 Now divide both sides by 2: $x = 6$

Answer: $x = 6$

4

[2 marks]

Solve $5x - 3 = 22$

1 Add 3 to both sides: $5x = 25$

2 Divide both sides by 5: $x = 5$

3 Check: $5 \times 5 - 3 = 22$ ✓

Answer: $x = 5$

5

[3 marks]

Solve $4x + 1 = 2x + 11$

1 Get all the x terms on one side. Subtract $2x$ from both sides.

2 $2x + 1 = 11$

3 Subtract 1 from both sides: $2x = 10$

4 Divide by 2: $x = 5$

Answer: $x = 5$

6

[3 marks]

Solve $3(x + 4) = 27$

1 The quickest route is to divide both sides by 3 first: $x + 4 = 9$

2 Subtract 4: $x = 5$

3 (Expanding first also works: $3x + 12 = 27$, so $3x = 15$ and $x = 5$.)

Answer: $x = 5$

7

[3 marks]

Solve $\frac{x + 5}{3} = 4$

1 Multiply both sides by 3 to clear the fraction: $x + 5 = 12$

2 Subtract 5 from both sides: $x = 7$

3 Check: $(7 + 5) \div 3 = 4$ ✓

Answer: $x = 7$

8

[3 marks]

Solve $7 - 2x = 1$

1 Add $2x$ to both sides so the x term becomes positive: $7 = 1 + 2x$

2 Subtract 1 from both sides: $6 = 2x$

3 Divide by 2: $x = 3$

4 Check: $7 - 2 \times 3 = 7 - 6 = 1$ ✓

Answer: $x = 3$

9

[4 marks]

The three angles of a triangle are x° , $2x^\circ$ and $(3x + 30)^\circ$.

(a) Form an equation in x and solve it.

(b) Work out the size of each angle.

1 (a) The angles of a triangle add up to 180° .

2 $x + 2x + 3x + 30 = 180$, which simplifies to $6x + 30 = 180$

3 $6x = 150$, so $x = 25$

4 (b) The angles are 25° , $2 \times 25 = 50^\circ$ and $3 \times 25 + 30 = 105^\circ$

5 Check: $25 + 50 + 105 = 180$ ✓

Answer: (a) $x = 25$ (b) 25° , 50° and 105°

10**[4 marks]**

A rectangle has length $(3x - 1)$ cm and width $(x + 2)$ cm.

The perimeter is 42 cm.

(a) Find the value of x .

(b) Work out the area of the rectangle.

1 (a) Perimeter = $2 \times (\text{length} + \text{width}) = 2[(3x - 1) + (x + 2)]$

2 Inside the bracket: $4x + 1$, so the perimeter is $8x + 2$

3 $8x + 2 = 42$, so $8x = 40$ and $x = 5$

4 (b) Length = $3 \times 5 - 1 = 14$ cm and width = $5 + 2 = 7$ cm

5 Area = $14 \times 7 = 98 \text{ cm}^2$ (check: perimeter $2(14 + 7) = 42 \checkmark$)

Answer: (a) $x = 5$ (b) 98 cm^2

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1**[2 marks]**

Solve $5x + 2 = 3x + 14$

1 Subtract $3x$ from both sides: $2x + 2 = 14$

2 Subtract 2: $2x = 12$, so $x = 6$

Answer: $x = 6$

2**[2 marks]**

Solve $2(3x - 1) = 4x + 8$

1 Expand the bracket: $6x - 2 = 4x + 8$

2 Subtract $4x$: $2x - 2 = 8$, so $2x = 10$ and $x = 5$

Answer: $x = 5$

3**[3 marks]**

Solve $\frac{2x - 1}{5} = \frac{x + 4}{3}$

1 Cross-multiply to clear both fractions: $3(2x - 1) = 5(x + 4)$

2 Expand both sides: $6x - 3 = 5x + 20$

3 Subtract $5x$: $x - 3 = 20$

4 $x = 23$

Answer: $x = 23$

4

[3 marks]

Solve $\frac{x}{2} + \frac{x}{3} = 10$

1 Multiply every term by 6, the lowest common multiple of 2 and 3.

2 $6 \times \frac{x}{2} = 3x$ and $6 \times \frac{x}{3} = 2x$

3 $3x + 2x = 60$, so $5x = 60$

4 $x = 12$ (check: $6 + 4 = 10$ ✓)

Answer: $x = 12$

5

[4 marks]

Solve $\frac{3}{x+1} = \frac{2}{x-3}$

1 Cross-multiply: $3(x-3) = 2(x+1)$

2 Expand: $3x - 9 = 2x + 2$

3 Subtract $2x$: $x - 9 = 2$

4 $x = 11$

5 Check: $\frac{3}{12} = 0.25$ and $\frac{2}{8} = 0.25$ ✓

Answer: $x = 11$

6

[4 marks]

Make l the subject of the formula

$$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$$

1 Divide both sides by 2π : $\frac{T}{2\pi} = \sqrt{\left(\frac{l}{g}\right)}$

2 Square both sides to remove the square root: $\frac{T^2}{4\pi^2} = \frac{l}{g}$

3 Remember that squaring 2π gives $4\pi^2$, not $2\pi^2$.

4 Multiply both sides by g .

Answer: $l = \frac{gT^2}{4\pi^2}$

7

[4 marks]

- (a) Show that the equation $x^2 - x - 5 = 0$ can be rearranged to give $x = \sqrt{5 + x}$
- (b) Starting with $x_0 = 2$, use the iteration $x_{n+1} = \sqrt{5 + x_n}$ three times. Give each value to 4 decimal places.

- 1 (a) Move the $-x$ and -5 across: $x^2 = x + 5$
- 2 Take the (positive) square root of both sides: $x = \sqrt{5 + x}$
- 3 (b) $x_1 = \sqrt{5 + 2} = \sqrt{7} = 2.6458$
- 4 $x_2 = \sqrt{5 + 2.6458} = \sqrt{7.6458} = 2.7651$
- 5 $x_3 = \sqrt{5 + 2.7651} = \sqrt{7.7651} = 2.7866$
- 6 The values are settling down towards the root, which is about 2.7913

Answer: (b) $x_1 = 2.6458$, $x_2 = 2.7651$, $x_3 = 2.7866$

8

[4 marks]

Solve $\frac{x+2}{4} - \frac{x-3}{6} = 1$

- 1 Multiply every term by 12, the lowest common multiple of 4 and 6.
- 2 $3(x+2) - 2(x-3) = 12$
- 3 Expand carefully — the minus sign multiplies both terms in the second bracket.
- 4 $3x + 6 - 2x + 6 = 12$
- 5 $x + 12 = 12$, so $x = 0$
- 6 Check: $\frac{2}{4} - \frac{-3}{6} = 0.5 + 0.5 = 1 \checkmark$

Answer: $x = 0$

9

[4 marks]

The three angles of a triangle are $(2x + 10)^\circ$, $(3x - 20)^\circ$ and $(x + 40)^\circ$.

- (a) Show that $6x + 30 = 180$
- (b) Work out the size of the largest angle.
- 1 (a) Add the three expressions: $2x + 3x + x = 6x$
 - 2 The numbers: $10 - 20 + 40 = 30$
 - 3 The angles of a triangle total 180° , so $6x + 30 = 180$
 - 4 (b) $6x = 150$, so $x = 25$
 - 5 The angles are $2(25) + 10 = 60^\circ$, $3(25) - 20 = 55^\circ$ and $25 + 40 = 65^\circ$
 - 6 Check: $60 + 55 + 65 = 180 \checkmark$, and the largest is 65°

Answer: (b) 65°

(a) Solve $\frac{2x+3}{3} + \frac{x-1}{2} = 4$

(b) When 5 is added to a number and the result is doubled, the answer is the same as multiplying the number by 3 and then subtracting 1. Find the number.

1 (a) Multiply every term by 6: $2(2x+3) + 3(x-1) = 24$

2 Expand: $4x + 6 + 3x - 3 = 24$

3 Collect: $7x + 3 = 24$, so $7x = 21$ and $x = 3$

4 (b) Call the number n . 'Add 5 then double' is $2(n+5)$.

5 'Multiply by 3 then subtract 1' is $3n - 1$, so $2(n+5) = 3n - 1$

6 $2n + 10 = 3n - 1$, so $n = 11$. Check: $2 \times 16 = 32$ and $3 \times 11 - 1 = 32$ ✓

Answer: (a) $x = 3$ (b) 11