Inequalities

GCSE Maths · Worked Solutions

20 questions · 62 marks total

jtmaths.org · free maths resources

Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 **[1 mark]**

List all the integers n that satisfy $-2 < n \leq 2$

1 The strict sign < means −2 itself is **not** included.

2 The sign ≤ means 2 **is** included.

Answer: −1, 0, 1, 2

2 **[2 marks]**

Solve $x + 5 > 9$

1 Subtract 5 from both sides, exactly as you would for an equation.

2 $x > 4$

Answer: $x > 4$

3 **[2 marks]**

Solve $3x \leq 21$

1 Divide both sides by 3.

2 Because 3 is positive, the inequality sign stays the same way round.

Answer: $x \leq 7$

4 **[2 marks]**

Solve $2x - 1 < 11$

1 Add 1 to both sides: $2x < 12$

2 Divide both sides by 2: $x < 6$

Answer: $x < 6$

5 **[3 marks]**

(a) Solve $5x + 3 \geq 23$

(b) Write down the smallest integer that satisfies the inequality.

1 **(a)** Subtract 3 from both sides: $5x \geq 20$

2 Divide by 5: $x \geq 4$

3 **(b)** The sign ≥ includes 4 itself, so the smallest integer is 4.

Answer: (a) $x \geq 4$ (b) 4

6 [3 marks]

Solve $4(x - 2) \leq 12$

1 Divide both sides by 4 first: $x - 2 \leq 3$

2 Add 2 to both sides: $x \leq 5$

3 (Expanding first also works: $4x - 8 \leq 12$, so $4x \leq 20$.)

Answer: $x \leq 5$

7 [3 marks]

Solve $-3 < 2x + 1 \leq 9$

1 Whatever you do, do it to all three parts of the inequality.

2 Subtract 1 throughout: $-4 < 2x \leq 8$

3 Divide throughout by 2: $-2 < x \leq 4$

Answer: $-2 < x \leq 4$

8 [3 marks]

Solve $7 - x > 2$

1 The safest method is to add x to both sides: $7 > 2 + x$

2 Subtract 2: $5 > x$, which is the same as $x < 5$

3 If instead you subtract 7 and get $-x > -5$, remember that dividing by -1 **flips** the sign to give $x < 5$

Answer: $x < 5$

9 [4 marks]

List all the integer values of n that satisfy $3 \leq 2n + 1 < 13$

1 Subtract 1 from all three parts: $2 \leq 2n < 12$

2 Divide all three parts by 2: $1 \leq n < 6$

3 n = 1 is included because of the ≤ sign.

4 n = 6 is **not** included because of the < sign.

Answer: n = 1, 2, 3, 4, 5

10 [4 marks]

A rectangle has width x cm and length $(x + 4)$ cm.

The perimeter is less than 40 cm, and x is a whole number greater than 3.

Find all the possible values of x.

1 Perimeter $= 2[x + (x + 4)] = 2(2x + 4) = 4x + 8$

2 So $4x + 8 < 40$

3 $4x < 32$, giving $x < 8$

4 We are also told $x > 3$, so $3 < x < 8$

5 The whole numbers strictly between 3 and 8 are 4, 5, 6 and 7.

Answer: $x = 4$, 5, 6 or 7

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1 **[2 marks]**

Solve $5 - 2x \geq 1$

1 Subtract 5 from both sides: $-2x \geq -4$

2 Divide both sides by -2. Dividing by a negative **reverses** the inequality sign.

3 $x \leq 2$

Answer: $x \leq 2$

2 **[2 marks]**

Solve $3(x + 2) < 5x - 4$

1 Expand: $3x + 6 < 5x - 4$

2 Subtract $3x$: $6 < 2x - 4$

3 Add 4: $10 < 2x$, so $5 < x$, i.e. $x > 5$

Answer: $x > 5$

3 **[3 marks]**

Solve $x^2 < 36$

1 Do not just write $x < 6$ — negative values matter too.

2 The critical values come from $x^2 = 36$, so $x = 6$ or $x = -6$

3 Sketch $y = x^2 - 36$: a U-shaped curve crossing at -6 and 6.

4 It is below zero **between** the two roots.

Answer: $-6 < x < 6$

4 **[3 marks]**

Solve $x^2 \geq 49$

1 The critical values are $x = 7$ and $x = -7$

2 The curve $y = x^2 - 49$ is above (or on) the axis **outside** the roots.

3 So the solution is in two separate pieces.

Answer: $x \leq -7$ or $x \geq 7$

5 **[4 marks]**

Solve $x^2 - 2x - 15 < 0$

1 Factorise to find the critical values: two numbers multiplying to -15 and adding to -2 are -5 and $+3$.

2 $(x - 5)(x + 3) = 0$ gives $x = 5$ and $x = -3$

3 Sketch the U-shaped curve crossing the axis at -3 and 5.

4 The curve is **below** the axis between the roots.

Answer: $-3 < x < 5$

6 **[4 marks]**

Solve $x^2 + 4x - 12 \geq 0$

1 Factorise: two numbers multiplying to −12 and adding to 4 are 6 and −2.

2 $(x + 6)(x - 2) = 0$, so the critical values are −6 and 2

3 The U-shaped curve is above or on the axis outside the roots.

4 Because the sign is ≥, the critical values themselves are included.

Answer: $x \leq -6$ or $x \geq 2$

7 **[4 marks]**

Solve $2x^2 - 5x - 3 \leq 0$

1 Multiply the end numbers: $2 \times (-3) = -6$. Two numbers multiplying to −6 and adding to −5 are −6 and +1.

2 $2x^2 - 6x + x - 3 = 2x(x - 3) + 1(x - 3) = (2x + 1)(x - 3)$

3 Critical values: $x = -\frac{1}{2}$ and $x = 3$

4 The coefficient of x^2 is positive, so the curve is below or on the axis between the roots.

Answer: $-\frac{1}{2} \leq x \leq 3$

8 **[4 marks]**

The region R is defined by the three inequalities

$y \geq 1$

$y \leq x + 2$

$x + y \leq 6$

Find the coordinates of the three corners of R.

1 The corners are where each pair of boundary lines cross.

2 $y = 1$ meets $y = x + 2$ when $1 = x + 2$, so $x = -1 \rightarrow (-1, 1)$

3 $y = 1$ meets $x + y = 6$ when $x + 1 = 6$, so $x = 5 \rightarrow (5, 1)$

4 $y = x + 2$ meets $x + y = 6$: substitute to get $x + x + 2 = 6$

5 $2x = 4$, so $x = 2$ and $y = 4 \rightarrow (2, 4)$

Answer: (−1, 1), (5, 1) and (2, 4)

9 **[4 marks]**

Kim has £50. She buys a bag costing £14 and wants to spend the rest on books costing £6.50 each. Form an inequality and use it to find the greatest number of books she can buy.

1 Let n be the number of books. The total spent must be at most £50.

2 $14 + 6.5n \leq 50$

3 Subtract 14: $6.5n \leq 36$

4 Divide by 6.5: $n \leq 5.538\ldots$

5 n must be a whole number, so the greatest possible value is 5.

6 Check: $14 + 5 \times 6.50 = £46.50$, and 6 books would cost £53 — too much.

Answer: $14 + 6.5n \leq 50$, giving 5 books

10 **[5 marks]**

Find all the integer values of x that satisfy **both** of these inequalities.

$x^2 - 16 < 0$

$2x + 5 \geq 1$

1 Solve each inequality separately, then find where the two solutions overlap.

2 First: $x^2 < 16$, with critical values $x = -4$ and $x = 4$

3 A U-shaped curve is below the axis between its roots, so $-4 < x < 4$

4 Second: $2x \geq -4$, so $x \geq -2$

5 The overlap is $-2 \leq x < 4$

6 The integers in that range are −2, −1, 0, 1, 2 and 3 (4 is excluded).

Answer: $x = -2, -1, 0, 1, 2, 3$