# Sequences

GCSE Maths · Worked Solutions

20 questions · 62 marks total

jtmaths.org · free maths resources

## Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

**1** **[1 mark]**

Write down the next term of this sequence.

**4 9 14 19**

1 The sequence goes up by 5 each time.

2 19 + 5 = 24

**Answer:** 24

**2** **[2 marks]**

Find an expression for the $n$th term of this sequence.

**3 7 11 15**

1 The common difference is 4, so the $n$th term starts with $4n$.

2 $4n$ gives 4, 8, 12, 16 — each is 1 more than the sequence.

3 So subtract 1.

**Answer:** $4n - 1$

**3** **[2 marks]**

The $n$th term of a sequence is $5n + 2$

Write down the first three terms.

1 Substitute $n = 1$: $5(1) + 2 = 7$

2 $n = 2$: $5(2) + 2 = 12$

3 $n = 3$: $5(3) + 2 = 17$

**Answer:** 7, 12, 17

**4** **[2 marks]**

Find the 10th term of the sequence

**6 10 14 18 …**

1 The common difference is 4, so the $n$th term is $4n + 2$

2 Substitute $n = 10$: $4(10) + 2$

3 = 42

**Answer:** 42

**5** [3 marks]

Here is a sequence.

**20 17 14 11**

(a) Write down the next two terms.

(b) Find an expression for the $n$th term.

(c) Is −7 a term of this sequence? Give a reason.

1 **(a)** The sequence goes down by 3 each time: 8 and 5

2 **(b)** A difference of −3 means the $n$th term contains $-3n$.

3 $-3n$ gives −3, −6, −9…, and the sequence is 23 more each time, so the $n$th term is $23 - 3n$

4 **(c)** Set $23 - 3n = -7$, so $3n = 30$ and $n = 10$

5 $n = 10$ is a whole number, so yes — −7 is the 10th term.

**Answer:** (a) 8 and 5 (b) $23 - 3n$ (c) Yes, it is the 10th term

**6** [3 marks]

Here is a sequence.

**2 5 7 12 19**

(a) Describe the rule for finding the next term.

(b) Write down the next two terms.

1 **(a)** Check whether each term is the sum of the two before it: 2 + 5 = 7 ✓, 5 + 7 = 12 ✓, 7 + 12 = 19 ✓

2 The rule is: add the two previous terms.

3 **(b)** 12 + 19 = 31

4 19 + 31 = 50

**Answer:** (a) add the previous two terms (b) 31 and 50

**7** [3 marks]

Here is a sequence.

**3 6 12 24**

(a) Describe the rule for finding the next term.

(b) Write down the next two terms.

(c) Work out the 7th term.

1 **(a)** Each term is double the one before, so the rule is ‘multiply by 2’.

2 **(b)** 24 × 2 = 48 and 48 × 2 = 96

3 **(c)** Continue: the 6th term is 96 and the 7th is 96 × 2 = 192

4 (Or use $3 \times 2^6 = 3 \times 64 = 192$.)

**Answer:** (a) multiply by 2 (b) 48 and 96 (c) 192

**8** [3 marks]

(a) Write down the first five square numbers.

(b) Write down the first five triangular numbers.

(c) Which number appears in both lists?

1 **(a)** Square numbers come from $1^2$, $2^2$, $3^2$…: 1, 4, 9, 16, 25

2 **(b)** Triangular numbers add 1, then 2, then 3…: 1, 3, 6, 10, 15

3 **(c)** The number 1 appears in both lists.

**Answer:** (a) 1, 4, 9, 16, 25 (b) 1, 3, 6, 10, 15 (c) 1

**9** [4 marks]

The $n$th term of a sequence is $3n + 4$

(a) Show that 102 is not a term of the sequence.

(b) Find the largest term of the sequence that is less than 200.

1 **(a)** Set $3n + 4 = 102$, so $3n = 98$

2 $n = 32.66…$, which is not a whole number, so 102 cannot be a term.

3 **(b)** Solve $3n + 4 < 200$, so $3n < 196$ and $n < 65.33…$

4 The largest whole number is $n = 65$

5 The term is $3(65) + 4 = 199$

**Answer:** (a) $3n = 98$ gives $n = 32.6…$, not a whole number (b) 199

**10** [4 marks]

Matchstick patterns are made as shown. Pattern 1 uses 4 matches, pattern 2 uses 7 matches and pattern 3 uses 10 matches.

(a) Find an expression for the number of matches in pattern $n$.

(b) How many matches are needed for pattern 20?

(c) Which pattern uses exactly 100 matches?

1 **(a)** The numbers 4, 7, 10 go up by 3, so the expression starts with $3n$.

2 $3n$ gives 3, 6, 9 — each is 1 less than the pattern, so the expression is $3n + 1$

3 **(b)** $3(20) + 1 = 61$ matches

4 **(c)** $3n + 1 = 100$, so $3n = 99$ and $n = 33$

**Answer:** (a) $3n + 1$ (b) 61 (c) pattern 33

## Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

**1** [2 marks]

Find an expression for the $n$th term of

**2 9 16 23**

1 The common difference is 7, so start with $7n$.

2 $7n$ gives 7, 14, 21 — each is 5 more than the sequence.

**Answer:** $7n - 5$

**2** [2 marks]

The $n$th term of a sequence is $n^2 + 3$

Work out the 6th term.

1 Substitute $n = 6$: $6^2 + 3$

2 $= 36 + 3 = 39$

**Answer:** 39

**3** [3 marks]

Find an expression for the $n$th term of the quadratic sequence

**4 7 12 19 28**

1 First differences: 3, 5, 7, 9. Second differences: 2, 2, 2 — constant, so it is quadratic.

2 Half of the second difference is 1, so the expression starts with $n^2$.

3 $n^2$ gives 1, 4, 9, 16, 25

4 Subtracting: $4 - 1 = 3$, $7 - 4 = 3$, $12 - 9 = 3$… a constant 3.

**Answer:** $n^2 + 3$

**4** [3 marks]

Find an expression for the $n$th term of

**5 12 23 38 57**

1 First differences: 7, 11, 15, 19. Second differences: 4, 4, 4.

2 Half of 4 is 2, so the expression starts with $2n^2$.

3 $2n^2$ gives 2, 8, 18, 32, 50

4 Subtracting: 3, 4, 5, 6, 7 — a linear sequence with $n$th term $n + 2$

5 Check $n = 3$: $2(9) + 3 + 2 = 23$ ✓

**Answer:** $2n^2 + n + 2$

**5** [4 marks]

Here is a geometric sequence.

**5 15 45 …**

(a) Write down the common ratio.

(b) Find an expression for the $n$th term.

(c) Work out the 8th term.

1 **(a)** $15 \div 5 = 3$ and $45 \div 15 = 3$, so the common ratio is 3.

2 **(b)** The first term is 5 and each step multiplies by 3.

3 The $n$th term is $5 \times 3^{n-1}$ (the power is $n - 1$ because the first term uses $3^0 = 1$).

4 **(c)** 8th term $= 5 \times 3^7$

5 $3^7 = 2187$, so the term is $5 \times 2187 = 10\,935$

**Answer:** (a) 3 (b) $5 \times 3^{n-1}$ (c) 10 935

**6** [4 marks]

The $n$th term of a sequence is $n^2 + 2n$

Show that 168 is a term of the sequence and state its position.

1. Set the expression equal to 168: $n^2 + 2n = 168$
2. Rearrange into a quadratic: $n^2 + 2n - 168 = 0$
3. Two numbers multiplying to $-168$ and adding to 2 are 14 and $-12$.
4. $(n + 14)(n - 12) = 0$, so $n = -14$ or $n = 12$
5. A position must be a positive whole number, so $n = 12$
6. Check: $12^2 + 2(12) = 144 + 24 = 168$ ✓

**Answer:** Yes — 168 is the 12th term

**7** [4 marks]

The $n$th term of a sequence is $5n - 3$

(a) Find the first term of the sequence that is greater than 500.

(b) Prove that every term of the sequence is 2 more than a multiple of 5.

1. **(a)** Solve $5n - 3 > 500$, so $5n > 503$ and $n > 100.6$
2. The first whole number is $n = 101$, giving $5(101) - 3 = 502$
3. **(b)** Rewrite the expression: $5n - 3 = 5n - 5 + 2$
4. $= 5(n - 1) + 2$
5. Since $n - 1$ is an integer, $5(n - 1)$ is a multiple of 5, so every term is 2 more than a multiple of 5.

**Answer:** (a) 502 (b) $5n - 3 = 5(n - 1) + 2$

**8** [4 marks]

A sequence is defined by $u_1 = 5$ and $u_{n+1} = 2u_n - 3$

(a) Work out $u_2$, $u_3$ and $u_4$.

(b) Describe what happens to the terms as $n$ increases.

1. **(a)** $u_2 = 2(5) - 3 = 7$
2. $u_3 = 2(7) - 3 = 11$
3. $u_4 = 2(11) - 3 = 19$
4. **(b)** The gaps between terms (2, 4, 8…) keep doubling, so the terms grow faster and faster and increase without limit.

**Answer:** (a) 7, 11, 19 (b) the terms increase, and the increases keep doubling, so the sequence grows without limit

**9** [4 marks]

A pattern of dots grows like this: 1, 5, 13, 25, 41, …

Find an expression for the number of dots in the $n$th pattern.

1 First differences: 4, 8, 12, 16. Second differences: 4, 4, 4 — so the sequence is quadratic.

2 Half of 4 is 2, so start with $2n^2$.

3 $2n^2$ gives 2, 8, 18, 32, 50

4 Subtract from the sequence: $1 - 2 = -1$, $5 - 8 = -3$, $13 - 18 = -5$, $25 - 32 = -7$

5 That leftover sequence −1, −3, −5, −7 has $n$th term $1 - 2n$

6 Check $n = 4$: $2(16) - 8 + 1 = 32 - 7 = 25$ ✓

**Answer:** $2n^2 - 2n + 1$

**10** [5 marks]

The $n$th term of a sequence is $n^2 + 5n$

(a) Prove that every term of the sequence is an even number.

(b) Two consecutive terms of the sequence differ by 26. Find these two terms.

1 **(a)** Factorise the expression: $n^2 + 5n = n(n + 5)$

2 If $n$ is even, then $n(n + 5)$ has an even factor, so the product is even.

3 If $n$ is odd, then $n + 5$ is even, so again the product is even.

4 Either way the product is even, so every term is an even number.

5 **(b)** The difference between consecutive terms is $[(n + 1)^2 + 5(n + 1)] - [n^2 + 5n]$

6 $= (n^2 + 2n + 1 + 5n + 5) - (n^2 + 5n) = 2n + 6$

7 Set $2n + 6 = 26$, so $n = 10$

8 The terms are $u_{10} = 100 + 50 = 150$ and $u_{11} = 121 + 55 = 176$ (and $176 - 150 = 26$ ✓)

**Answer:** (a) $n(n + 5)$ always has an even factor (b) 150 and 176