

Simultaneous Equations

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 [1 mark]

$$x + y = 10 \text{ and } x = 4$$

Work out the value of y .

1 Substitute $x = 4$ into the first equation: $4 + y = 10$

2 $y = 6$

Answer: $y = 6$

2 [2 marks]

Solve

$$x + y = 9$$

$$x - y = 3$$

1 The y terms are $+y$ and $-y$, so **add** the equations.

2 $2x = 12$, so $x = 6$

3 Substitute back: $6 + y = 9$, so $y = 3$

Answer: $x = 6, y = 3$

3 [2 marks]

Solve

$$3x + y = 17$$

$$x + y = 7$$

1 Both equations have $+y$, so **subtract** the second from the first.

2 $2x = 10$, so $x = 5$

3 Substitute: $5 + y = 7$, so $y = 2$

Answer: $x = 5, y = 2$

4 [2 marks]

Solve

$$4x + y = 14$$

$$2x + y = 8$$

1 Subtract the second equation from the first to remove y .

2 $2x = 6$, so $x = 3$

3 Substitute into $2x + y = 8$: $6 + y = 8$, so $y = 2$

Answer: $x = 3, y = 2$

5

[3 marks]

Solve

$$2x + 3y = 16$$

$$2x + y = 8$$

1 The x terms match, so subtract the second equation from the first.

2 $3y - y = 2y$ and $16 - 8 = 8$, so $2y = 8$

3 $y = 4$

4 Substitute into $2x + y = 8$: $2x + 4 = 8$, so $x = 2$

Answer: $x = 2, y = 4$

6

[3 marks]

Solve

$$5x - 2y = 4$$

$$3x + 2y = 12$$

1 The y terms are $-2y$ and $+2y$, so **add** the equations.

2 $8x = 16$, so $x = 2$

3 Substitute into $3x + 2y = 12$: $6 + 2y = 12$

4 $2y = 6$, so $y = 3$

Answer: $x = 2, y = 3$

7

[3 marks]

Solve

$$x + 3y = 14$$

$$2x + y = 13$$

1 Neither pair matches, so multiply the first equation by 2 to match the x terms.

2 $2x + 6y = 28$

3 Subtract the second equation: $5y = 15$, so $y = 3$

4 Substitute into $x + 3y = 14$: $x + 9 = 14$, so $x = 5$

5 Check in the other equation: $2(5) + 3 = 13$ ✓

Answer: $x = 5, y = 3$

8

[3 marks]

Solve

$$y = 2x + 1$$

$$3x + y = 16$$

1 The first equation already gives y , so substitute it into the second.

2 $3x + (2x + 1) = 16$

3 $5x + 1 = 16$, so $5x = 15$ and $x = 3$

4 $y = 2(3) + 1 = 7$

Answer: $x = 3, y = 7$

9

[4 marks]

3 coffees and 2 teas cost £10.40

1 coffee and 2 teas cost £6.00

Work out the cost of one coffee and the cost of one tea.

1 Let c be the cost of a coffee and t the cost of a tea.

2 $3c + 2t = 10.40$ and $c + 2t = 6.00$

3 Both have $+2t$, so subtract: $2c = 4.40$

4 $c = £2.20$

5 Substitute: $2.20 + 2t = 6.00$, so $2t = 3.80$ and $t = £1.90$

6 Check: $3(2.20) + 2(1.90) = 6.60 + 3.80 = £10.40$ ✓

Answer: coffee £2.20, tea £1.90

10

[4 marks]

At a cinema, 2 adult tickets and 3 child tickets cost £34.50

3 adult tickets and 2 child tickets cost £38.00

Work out the cost of one adult ticket and one child ticket.

1 Let a be the adult price and c the child price.

2 $2a + 3c = 34.50$ and $3a + 2c = 38.00$

3 Multiply the first by 3 and the second by 2 to match the a terms.

4 $6a + 9c = 103.50$ and $6a + 4c = 76.00$

5 Subtract: $5c = 27.50$, so $c = £5.50$

6 Substitute: $2a + 16.50 = 34.50$, so $2a = 18$ and $a = £9.00$

Answer: adult £9.00, child £5.50

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1

[2 marks]

Solve

$$3x + 2y = 12$$

$$x - 2y = 4$$

1 Add the equations to eliminate y : $4x = 16$

2 $x = 4$

3 Substitute: $4 - 2y = 4$, so $y = 0$

Answer: $x = 4$, $y = 0$

2**[2 marks]**

Solve

$$y = 3x - 2$$

$$y = x + 6$$

1 Both right-hand sides equal y , so set them equal: $3x - 2 = x + 6$

2 $2x = 8$, so $x = 4$

3 $y = 4 + 6 = 10$

Answer: $x = 4, y = 10$

3**[3 marks]**

Solve

$$4x + 3y = 25$$

$$3x - 2y = 6$$

1 Multiply the first by 2 and the second by 3 so the y terms become $6y$ and $-6y$.

2 $8x + 6y = 50$ and $9x - 6y = 18$

3 Add them: $17x = 68$, so $x = 4$

4 Substitute into $4x + 3y = 25$: $16 + 3y = 25$, so $y = 3$

Answer: $x = 4, y = 3$

4**[3 marks]**

Solve

$$5x + 4y = 22$$

$$3x - 2y = 0$$

1 Multiply the second equation by 2: $6x - 4y = 0$

2 Add to the first: $11x = 22$, so $x = 2$

3 Substitute into $3x - 2y = 0$: $6 = 2y$, so $y = 3$

4 Check: $5(2) + 4(3) = 10 + 12 = 22$ ✓

Answer: $x = 2, y = 3$

5**[4 marks]**

Solve

$$y = x^2 + 2$$

$$y = 3x + 6$$

1 Set the two expressions for y equal: $x^2 + 2 = 3x + 6$

2 Rearrange: $x^2 - 3x - 4 = 0$

3 Factorise: $(x - 4)(x + 1) = 0$, so $x = 4$ or $x = -1$

4 Use the linear equation for the y values.

5 $x = 4$ gives $y = 18$, and $x = -1$ gives $y = 3$

Answer: $x = 4, y = 18$ and $x = -1, y = 3$

6

[4 marks]

Solve

$$x^2 + y^2 = 25$$

$$y = x + 1$$

- 1 Substitute $y = x + 1$ into the first equation.
- 2 $x^2 + (x + 1)^2 = 25$, and $(x + 1)^2 = x^2 + 2x + 1$
- 3 $2x^2 + 2x + 1 = 25$, so $2x^2 + 2x - 24 = 0$
- 4 Divide by 2: $x^2 + x - 12 = 0$, which factorises as $(x + 4)(x - 3) = 0$
- 5 $x = -4$ gives $y = -3$, and $x = 3$ gives $y = 4$
- 6 Check (3, 4): $9 + 16 = 25$ ✓

Answer: $x = 3, y = 4$ and $x = -4, y = -3$

7

[4 marks]

4 small boxes and 3 large boxes have a total mass of 6.9 kg.

6 small boxes and 2 large boxes have a total mass of 7.6 kg.

Work out the mass of one small box and one large box.

- 1 Let s and l be the masses in kilograms.
- 2 $4s + 3l = 6.9$ and $6s + 2l = 7.6$
- 3 Multiply the first by 2 and the second by 3 to match the l terms.
- 4 $8s + 6l = 13.8$ and $18s + 6l = 22.8$
- 5 Subtract: $10s = 9$, so $s = 0.9$ kg
- 6 Substitute: $3.6 + 3l = 6.9$, so $3l = 3.3$ and $l = 1.1$ kg

Answer: small box 0.9 kg, large box 1.1 kg

8

[4 marks]

Find the coordinates of the point where the line $y = 2x + 3$ crosses the line $3x + 2y = 20$

- 1 The crossing point satisfies both equations, so solve them simultaneously.
- 2 Substitute $y = 2x + 3$ into the second equation.
- 3 $3x + 2(2x + 3) = 20$
- 4 $3x + 4x + 6 = 20$, so $7x = 14$ and $x = 2$
- 5 $y = 2(2) + 3 = 7$
- 6 Check: $3(2) + 2(7) = 6 + 14 = 20$ ✓

Answer: (2, 7)

9

[4 marks]

Show that the line $y = x + 5$ is a tangent to the curve $y = x^2 + 3x + 6$, and find the point where they touch.

- 1 Set the two expressions equal: $x^2 + 3x + 6 = x + 5$
- 2 Rearrange: $x^2 + 2x + 1 = 0$
- 3 Factorise: $(x + 1)^2 = 0$
- 4 There is only one (repeated) solution, $x = -1$, so the line meets the curve at exactly one point — that is what makes it a tangent.
- 5 $y = -1 + 5 = 4$, so they touch at $(-1, 4)$

Answer: Repeated root $x = -1$, so it is a tangent, touching at $(-1, 4)$

10

[5 marks]

At a theatre, a group of 5 adults and 3 children pay £55.

A group of 3 adults and 7 children pay £59.

(a) Work out the price of an adult ticket and a child ticket.

(b) Work out the cost for 4 adults and 6 children.

- 1 **(a)** Let a and c be the ticket prices in pounds.
- 2 $5a + 3c = 55$ and $3a + 7c = 59$
- 3 Multiply the first by 7 and the second by 3 to match the c terms.
- 4 $35a + 21c = 385$ and $9a + 21c = 177$
- 5 Subtract: $26a = 208$, so $a = £8$
- 6 Substitute into $5a + 3c = 55$: $40 + 3c = 55$, so $3c = 15$ and $c = £5$
- 7 **(b)** $4 \times 8 + 6 \times 5 = 32 + 30 = £62$

Answer: (a) adult £8, child £5 (b) £62