Graphs

GCSE Maths · Worked Solutions

20 questions · 63 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 **[1 mark]**

Write down the gradient of the line with equation $y = 3x - 5$

1 A straight line written as $y = mx + c$ has gradient m.

2 Here the number multiplying x is 3.

Answer: 3

2 **[2 marks]**

A straight line has equation $y = 2x + 1$

(a) Write down the gradient of the line.

(b) Write down the coordinates of the point where the line crosses the y-axis.

1 Compare with $y = mx + c$, where m is the gradient and c is the y-intercept.

2 **(a)** m = 2

3 **(b)** c = 1, and the line crosses the y-axis where $x = 0$, so the point is (0, 1)

Answer: (a) 2 (b) (0, 1)

3 **[2 marks]**

Does the point (3, 7) lie on the line $y = 2x + 1$?

You must show your working.

1 Substitute the x-coordinate into the equation and see whether you get the y-coordinate.

2 $x = 3$ gives $y = 2 \times 3 + 1 = 7$

3 This matches the y-coordinate of the point, so (3, 7) is on the line.

Answer: Yes — when $x = 3$, $y = 2(3) + 1 = 7$

4 **[3 marks]**

Complete the table of values for $y = 2x - 3$

x	−1	0	1	2	3
y		−3		1	

1 Substitute each x-value into $y = 2x - 3$ in turn.

2 $x = -1$: $y = 2(-1) - 3 = -2 - 3 = -5$

3 $x = 1$: $y = 2(1) - 3 = -1$

4 $x = 3$: $y = 2(3) - 3 = 3$

5 Sensible check: the y-values go up by 2 each time, matching the gradient of 2.

Answer: −5, −3, −1, 1, 3

5 **[3 marks]**

A is the point (1, 2) and B is the point (7, 10).

(a) Write down the coordinates of the midpoint of AB.

(b) Work out the gradient of the line AB.

1 **(a)** The midpoint is the average of the coordinates.

2 x: (1 + 7) ÷ 2 = 4 and y: (2 + 10) ÷ 2 = 6, so the midpoint is (4, 6)

3 **(b)** Gradient = $\dfrac{\text{change in } y}{\text{change in } x}$

4 Change in y = 10 − 2 = 8 and change in x = 7 − 1 = 6

5 Gradient = $\dfrac{8}{6} = \dfrac{4}{3}$

Answer: (a) (4, 6) (b) $\dfrac{4}{3}$

6 **[3 marks]**

(a) Find the equation of the line with gradient −2 that passes through (0, 4).

(b) Is your line parallel to $y = -2x + 9$? Give a reason.

1 **(a)** Use $y = mx + c$ with m = −2.

2 The point (0, 4) is on the y-axis, so it gives the intercept directly: c = 4

3 The equation is $y = -2x + 4$

4 **(b)** Parallel lines have equal gradients.

5 Both lines have gradient −2, so yes, they are parallel (they just cross the y-axis at different points).

Answer: (a) $y = -2x + 4$ (b) Yes — both have gradient −2

7 **[3 marks]**

A taxi firm charges a fixed fee of £3 plus £2 for every mile travelled.

(a) Write a formula for the cost C pounds of a journey of m miles.

(b) Work out the cost of a 7 mile journey.

(c) If the formula is drawn as a graph, what does the number 3 tell you?

1 **(a)** The mileage part is 2 × m and the fixed fee is added on: C = 2m + 3

2 **(b)** Substitute m = 7: C = 2 × 7 + 3 = 14 + 3 = 17

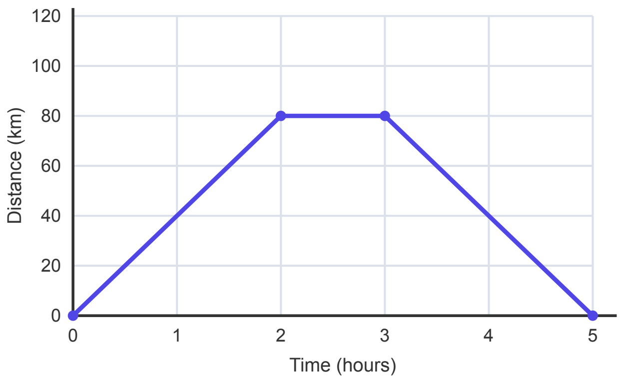
3 So the journey costs £17

4 **(c)** The 3 is the value of C when m = 0, so it is the y-intercept of the graph — the fixed charge you pay before travelling any distance.

Answer: (a) C = 2m + 3 (b) £17 (c) it is the y-intercept — the fixed charge

8 **[3 marks]**

The distance–time graph shows a cyclist's journey away from home and back again.



(a) How far had the cyclist travelled after 2 hours?

(b) For how long did the cyclist stop?

(c) Work out the cyclist's speed during the first stage of the journey.

1. **(a)** Read up from 2 hours to the graph, then across to the distance axis: 80 km
2. **(b)** A horizontal section means the distance is not changing, so the cyclist is stopped.
3. The flat part runs from 2 hours to 3 hours, so the stop lasted 1 hour.
4. **(c)** On a distance–time graph, speed is the gradient.
5. Speed = $\frac{80 \text{ km}}{2 \text{ hours}}$ = 40 km/h

Answer: (a) 80 km (b) 1 hour (c) 40 km/h

9 **[4 marks]**

A straight line L passes through the points (1, 5) and (4, 14).

Find the equation of L.

1. First find the gradient: $m = \frac{14 - 5}{4 - 1} = \frac{9}{3} = 3$
2. Now use $y = 3x + c$ and substitute one of the points.
3. Using (1, 5): $5 = 3 \times 1 + c$
4. So $c = 5 - 3 = 2$
5. The equation is $y = 3x + 2$
6. Check with the other point: $3 \times 4 + 2 = 14$ ✓

Answer: $y = 3x + 2$

10 **[4 marks]**

Two mobile phone tariffs are available.

Plan A: £10 per month plus 5p for each minute of calls.

Plan B: no monthly fee, 15p for each minute of calls.

(a) Write a formula for the monthly cost, in pounds, of each plan for m minutes of calls.

(b) Find the number of minutes for which the two plans cost the same.

(c) Which plan is cheaper for someone who talks for 150 minutes a month? Show your working.

1. **(a)** Work in pounds, so 5p = £0.05 and 15p = £0.15
2. Plan A: C = 0.05m + 10   and   Plan B: C = 0.15m
3. **(b)** The plans cost the same where the two graphs cross, so set the formulas equal: 0.05m + 10 = 0.15m
4. Subtract 0.05m from both sides: 10 = 0.1m, so m = 100 minutes
5. **(c)** Plan A for 150 minutes: 0.05 × 150 + 10 = 7.50 + 10 = £17.50
6. Plan B for 150 minutes: 0.15 × 150 = £22.50, so Plan A is cheaper by £5.

Answer: (a) A: C = 0.05m + 10, B: C = 0.15m   (b) 100 minutes   (c) Plan A (£17.50 against £22.50)

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1 **[2 marks]**

Work out the gradient of the line joining (−2, 1) and (4, 13).

1. Gradient = $\frac{\text{change in } y}{\text{change in } x}$
2. Change in y = 13 − 1 = 12 and change in x = 4 − (−2) = 6
3. Gradient = 12 ÷ 6 = 2

Answer: 2

2 **[2 marks]**

Rearrange $2y - 6x = 8$ into the form $y = mx + c$ and write down the gradient.

1. Add $6x$ to both sides: $2y = 6x + 8$
2. Divide every term by 2: $y = 3x + 4$
3. The gradient is the number multiplying x, which is 3.

Answer: $y = 3x + 4$, gradient 3

3 **[3 marks]**

Find the equation of the line that is parallel to $y = 3x - 1$ and passes through the point (2, 11).

1. Parallel lines have the same gradient, so the new line is $y = 3x + c$
2. Substitute the point (2, 11): 11 = 3 × 2 + c
3. 11 = 6 + c, so c = 5
4. The equation is $y = 3x + 5$

Answer: $y = 3x + 5$

4 **[3 marks]**

Find the equation of the line perpendicular to $y = 2x + 3$ that passes through the point (4, 1).

1 Perpendicular gradients multiply to −1, so the new gradient is $-\frac{1}{2}$

2 The line is $y = -\frac{1}{2}x + c$

3 Substitute (4, 1): $1 = -\frac{1}{2} \times 4 + c = -2 + c$

4 So c = 3 and the equation is $y = -\frac{1}{2}x + 3$

Answer: $y = -\frac{1}{2}x + 3$ (or $x + 2y = 6$)

5 **[4 marks]**

The value, £V, of a car after t years is modelled by

$V = 12000 \times 0.85^t$

(a) Write down the value of the car when it was new.

(b) Work out the value of the car after 3 years.

(c) What does the number 0.85 tell you about the value of the car?

1 **(a)** 'When new' means t = 0, and $0.85^0 = 1$, so V = 12000

2 **(b)** Substitute t = 3: $V = 12000 \times 0.85^3$

3 $0.85^3 = 0.614125$

4 V = 12000 × 0.614125 = 7369.5, so the car is worth £7369.50

5 **(c)** Each year the value is multiplied by 0.85, which is 85% of the previous year's value.

6 So the car loses 15% of its value every year — this is exponential decay, giving a curve that falls steeply then flattens.

Answer: (a) £12 000 (b) £7369.50 (c) the value falls by 15% each year

6 **[4 marks]**

The graph of $y = f(x)$ has a minimum point at (2, −3).

Write down the coordinates of the minimum point of each of these graphs.

(a) $y = f(x) + 5$

(b) $y = f(x - 3)$

(c) $y = -f(x)$

(d) $y = f(2x)$

1 **(a)** Adding 5 *outside* the function shifts the graph up 5, so only the y-coordinate changes: (2, 2)

2 **(b)** Changing x to $x - 3$ *inside* the function shifts the graph 3 to the **right**: (5, −3)

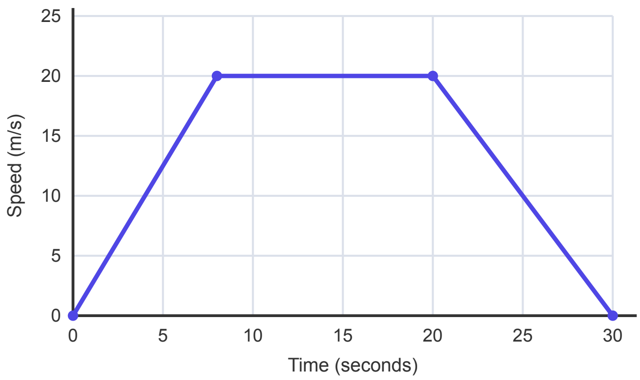
3 **(c)** The minus in front reflects the graph in the x-axis, so the y-coordinate changes sign: (2, 3). (This point is now a maximum.)

4 **(d)** $f(2x)$ is a stretch parallel to the x-axis with scale factor $\frac{1}{2}$, so the x-coordinate is halved: (1, −3)

Answer: (a) (2, 2) (b) (5, −3) (c) (2, 3) (d) (1, −3)

7 **[4 marks]**

The speed–time graph shows the first 30 seconds of a car's journey.



(a) Work out the acceleration during the first 8 seconds.

(b) Work out the total distance travelled in the 30 seconds.

1 **(a)** On a speed–time graph the gradient is the acceleration.

2 The speed rises from 0 to 20 m/s in 8 s, so acceleration = $\frac{20}{8}$ = 2.5 m/s^2

3 **(b)** On a speed–time graph the **area under the graph** is the distance travelled.

4 Split the shape into a triangle, a rectangle and a triangle.

5 Triangle (0–8 s): $\frac{1}{2} \times 8 \times 20 = 80$ m

6 Rectangle (8–20 s): $12 \times 20 = 240$ m · Triangle (20–30 s): $\frac{1}{2} \times 10 \times 20 = 100$ m

7 Total = 80 + 240 + 100 = 420 m

> **Answer:** (a) 2.5 m/s^2 (b) 420 m

8 **[4 marks]**

(a) Factorise fully $x^3 - 4x$

(b) Hence write down the coordinates of the three points where the curve $y = x^3 - 4x$ crosses the x-axis.

1 **(a)** Every term has a factor of x: $x(x^2 - 4)$

2 The bracket is the difference of two squares: $x^2 - 4 = (x + 2)(x - 2)$

3 So $x^3 - 4x = x(x + 2)(x - 2)$

4 **(b)** The curve meets the x-axis where $y = 0$, so each factor in turn is zero.

5 $x = 0$, $x = -2$ and $x = 2$

> **Answer:** (a) $x(x + 2)(x - 2)$ (b) $(-2, 0)$, $(0, 0)$ and $(2, 0)$

9 **[4 marks]**

The line $3x + 4y = 24$ crosses the x-axis at A and the y-axis at B. O is the origin.

(a) Write down the coordinates of A and of B.

(b) Work out the length of AB.

(c) Work out the area of triangle OAB.

1 **(a)** On the x-axis, $y = 0$: $3x = 24$, so $x = 8$ and A is (8, 0)

2 On the y-axis, $x = 0$: $4y = 24$, so $y = 6$ and B is (0, 6)

3 **(b)** OAB is right-angled at O, so use Pythagoras: $AB^2 = 8^2 + 6^2 = 64 + 36 = 100$

4 $AB = \sqrt{100} = 10$

5 **(c)** Area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 8 \times 6 = 24$ square units

Answer: (a) A(8, 0), B(0, 6) (b) 10 (c) 24 square units

10 **[5 marks]**

A circle has equation $x^2 + y^2 = 20$

The line $y = x - 2$ cuts the circle at the points P and Q.

(a) Find the coordinates of P and Q.

(b) Work out the exact length of PQ, giving your answer in the form $a\sqrt{b}$.

1 **(a)** Substitute $y = x - 2$ into the circle equation.

2 $x^2 + (x - 2)^2 = 20$, and $(x - 2)^2 = x^2 - 4x + 4$

3 So $2x^2 - 4x + 4 = 20$, giving $2x^2 - 4x - 16 = 0$

4 Divide by 2: $x^2 - 2x - 8 = 0$, which factorises to $(x - 4)(x + 2) = 0$

5 $x = 4$ gives $y = 4 - 2 = 2$, and $x = -2$ gives $y = -2 - 2 = -4$

6 So the points are (4, 2) and (−2, −4). Check (4, 2): 16 + 4 = 20 ✓

7 **(b)** Horizontal difference $= 4 - (-2) = 6$ and vertical difference $= 2 - (-4) = 6$

8 $PQ^2 = 6^2 + 6^2 = 72$, so $PQ = \sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

Answer: (a) P(4, 2) and Q(−2, −4) (b) $PQ = 6\sqrt{2}$