Quadratic and Other Graphs

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 **[1 mark]**

What is the name of the shape of the graph of $y = x^2$?

1 A quadratic graph is a smooth symmetrical U-shape.

2 Its mathematical name is a parabola.

Answer: a parabola (a U-shaped curve)

2 **[2 marks]**

Complete the table of values for $y = x^2$

x	−2	−1	0	1	2
y					

1 Squaring a negative number gives a positive answer.

2 $(-2)^2 = 4$ and $(-1)^2 = 1$

3 $0^2 = 0$, $1^2 = 1$ and $2^2 = 4$

4 Notice the symmetry of the values either side of 0.

Answer: 4, 1, 0, 1, 4

3 **[2 marks]**

Work out the values of y for $y = x^2 - 3$ when

(a) $x = -2$ (b) $x = 0$ (c) $x = 3$

1 **(a)** $(-2)^2 - 3 = 4 - 3 = 1$

2 **(b)** $0 - 3 = -3$

3 **(c)** $9 - 3 = 6$

Answer: (a) 1 (b) −3 (c) 6

4 **[2 marks]**

Write down the coordinates of the points where the curve $y = x^2 - 4$ crosses the x-axis.

1 On the x-axis, $y = 0$, so $x^2 - 4 = 0$

2 $x^2 = 4$, so $x = 2$ or $x = -2$

Answer: (2, 0) and (−2, 0)

5 **[3 marks]**

A curve has equation $y = x^2 - 2x - 3$

(a) Factorise $x^2 - 2x - 3$

(b) Write down where the curve crosses the x-axis.

(c) Write down where the curve crosses the y-axis.

1 **(a)** Two numbers multiplying to −3 and adding to −2 are −3 and +1.

2 So the answer is $(x - 3)(x + 1)$

3 **(b)** Setting each bracket to zero gives $x = 3$ and $x = -1$

4 The crossing points are (3, 0) and (−1, 0).

5 **(c)** Setting $x = 0$ gives $y = -3$, so (0, −3)

Answer: (a) $(x - 3)(x + 1)$ (b) (3, 0) and (−1, 0) (c) (0, −3)

6 **[3 marks]**

The curve $y = x^2 - 6x + 8$ crosses the x-axis at $x = 2$ and $x = 4$.

(a) Write down the equation of the line of symmetry.

(b) Work out the coordinates of the turning point.

1 **(a)** A parabola is symmetrical, so the line of symmetry is halfway between the roots.

2 (2 + 4) ÷ 2 = 3, so the line is $x = 3$

3 **(b)** The turning point lies on that line, so substitute $x = 3$.

4 $y = 9 - 18 + 8 = -1$, giving the minimum point (3, −1)

Answer: (a) $x = 3$ (b) (3, −1)

7 **[3 marks]**

Describe the shape of the graph of each equation.

(a) $y = x^3$

(b) $y = \dfrac{1}{x}$

(c) $y = 2^x$

1 **(a)** A cubic graph — a smooth curve with an S-shaped bend, going from bottom left to top right.

2 **(b)** A reciprocal graph — two separate curved branches, one in each of the top-right and bottom-left regions, never touching the axes.

3 **(c)** An exponential graph — always positive, rising more and more steeply, and getting closer and closer to the x-axis on the left without ever reaching it.

Answer: (a) cubic (b) reciprocal (two branches) (c) exponential growth

8 **[3 marks]**

(a) Complete the table of values for $y = x^2 + 2x$

x	−3	−2	−1	0	1
y					

(b) Write down the coordinates of the minimum point.

1 **(a)** $x = -3$: $9 - 6 = 3$ · $x = -2$: $4 - 4 = 0$

2 $x = -1$: $1 - 2 = -1$ · $x = 0$: 0 · $x = 1$: $1 + 2 = 3$

3 **(b)** The smallest y-value in the table is −1 at $x = -1$, and the values are symmetrical about it.

4 So the minimum point is (−1, −1).

Answer: (a) 3, 0, −1, 0, 3 (b) (−1, −1)

9 **[4 marks]**

Use an algebraic method to find the solutions of $x^2 - 2x - 4 = 0$, correct to 1 decimal place.

These are the values where the curve $y = x^2 - 2x - 4$ crosses the x-axis.

1 The expression will not factorise, so complete the square.

2 Half of −2 is −1: $(x - 1)^2 - 1 - 4 = (x - 1)^2 - 5$

3 $(x - 1)^2 = 5$, so $x - 1 = \pm\sqrt{5} = \pm 2.2360\ldots$

4 $x = 1 + 2.236 = 3.236\ldots$ or $x = 1 - 2.236 = -1.236\ldots$

5 So the curve crosses the axis at about $x = 3.2$ and $x = -1.2$

Answer: $x = 3.2$ and $x = -1.2$ (1 d.p.)

10 **[4 marks]**

A curve has equation $y = x^3 - 4x$

(a) Factorise $x^3 - 4x$ fully.

(b) Write down the three values of x where the curve meets the x-axis.

(c) Work out y when $x = 3$

1 **(a)** Take out the common factor x: $x(x^2 - 4)$

2 The bracket is a difference of two squares: $x(x + 2)(x - 2)$

3 **(b)** Setting each factor to zero gives $x = 0$, $x = -2$ and $x = 2$

4 **(c)** $y = 3^3 - 4 \times 3 = 27 - 12 = 15$

Answer: (a) $x(x + 2)(x - 2)$ (b) −2, 0 and 2 (c) 15

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1 **[2 marks]**

Write down the coordinates of the turning point of $y = (x - 2)^2 + 5$

1 The squared bracket is smallest (zero) when $x = 2$

2 Then $y = 5$, so the minimum point is (2, 5).

Answer: (2, 5)

2 **[2 marks]**

How many times does the curve $y = x^3 - 3x$ cross the x-axis? Show your working.

1 Set $y = 0$ and factorise: $x(x^2 - 3) = 0$

2 $x = 0$, or $x^2 = 3$ giving $x = \sqrt{3}$ and $x = -\sqrt{3}$

3 That is three different values.

Answer: 3 times, at $x = 0$, $\sqrt{3}$ and $-\sqrt{3}$

3 **[3 marks]**

Find the coordinates of the turning point of $y = x^2 + 4x - 1$ by completing the square.

1 Half of 4 is 2, so $(x + 2)^2 = x^2 + 4x + 4$

2 $y = (x + 2)^2 - 4 - 1 = (x + 2)^2 - 5$

3 The bracket is zero when $x = -2$, and then $y = -5$

Answer: minimum at $(-2, -5)$

4 **[3 marks]**

A circle has equation $x^2 + y^2 = 49$

(a) Write down the centre and the radius.

(b) Does the point (5, 5) lie inside, on, or outside the circle? Show your working.

1 **(a)** The form $x^2 + y^2 = r^2$ has centre at the origin (0, 0) and radius r.

2 $49 = 7^2$, so the radius is 7.

3 **(b)** Work out $x^2 + y^2$ for the point: $5^2 + 5^2 = 50$

4 50 is greater than 49, so the point is further than 7 from the centre — it lies outside.

Answer: (a) centre (0, 0), radius 7 (b) outside, since 50 > 49

5 **[4 marks]**

The point P(3, 4) lies on the circle $x^2 + y^2 = 25$

Find the equation of the tangent to the circle at P.

1 The tangent is perpendicular to the radius drawn to P.

2 Gradient of the radius OP = $\frac{4 - 0}{3 - 0} = \frac{4}{3}$

3 So the tangent gradient is $-\frac{3}{4}$

4 Use $y = -\frac{3}{4}x + c$ with P(3, 4): $4 = -2.25 + c$

5 $c = 6.25$, so $y = -\frac{3}{4}x + 6.25$

6 Multiplying by 4 gives the neater form $3x + 4y = 25$

Answer: $3x + 4y = 25$

6 **[4 marks]**

The graph of $y = f(x)$ passes through the point (2, 5).

Write down the coordinates of the corresponding point on each of these graphs.

(a) $y = f(x) - 3$ (b) $y = f(x + 1)$

(c) $y = 2f(x)$ (d) $y = f(-x)$

1 **(a)** Subtracting 3 outside the function moves the graph down 3: (2, 2)

2 **(b)** Replacing x by $x + 1$ inside moves the graph 1 to the **left**: (1, 5)

3 **(c)** Multiplying outside by 2 stretches vertically, doubling the y-coordinate: (2, 10)

4 **(d)** Replacing x by $-x$ reflects the graph in the y-axis: (−2, 5)

> **Answer:** (a) (2, 2) (b) (1, 5) (c) (2, 10) (d) (−2, 5)

7 **[4 marks]**

A curve has equation $y = 3 \times 2^x$

(a) Work out y when $x = 0$

(b) Work out y when $x = 4$

(c) Work out y when $x = -1$

(d) Explain why the curve never touches the x-axis.

1 **(a)** $2^0 = 1$, so $y = 3 \times 1 = 3$

2 **(b)** $2^4 = 16$, so $y = 3 \times 16 = 48$

3 **(c)** $2^{-1} = \frac{1}{2}$, so $y = 3 \times 0.5 = 1.5$

4 **(d)** 2^x is always greater than zero, however negative x becomes.

5 So y gets closer and closer to 0 but never actually reaches it — the x-axis is an asymptote.

> **Answer:** (a) 3 (b) 48 (c) 1.5 (d) 2^x is never zero

8 **[4 marks]**

A curve has equation $y = \frac{12}{x}$

(a) Complete the table of values.

x	1	2	3	4	6
y					

(b) Explain why $x = 0$ cannot be used.

(c) Describe the symmetry of the full graph.

1 **(a)** 12 ÷ 1 = 12, 12 ÷ 2 = 6, 12 ÷ 3 = 4, 12 ÷ 4 = 3, 12 ÷ 6 = 2

2 **(b)** Substituting $x = 0$ would mean dividing 12 by zero, which has no value.

3 So the curve has a break there and never crosses the y-axis.

4 **(c)** There are two branches, one for positive x and one for negative x.

5 The graph has rotational symmetry of order 2 about the origin.

> **Answer:** (a) 12, 6, 4, 3, 2 (b) it would mean dividing by zero (c) rotational symmetry of order 2 about the origin

9 **[4 marks]**

The curve $y = x^2 + 1$ passes through the points (1.9, 4.61) and (2.1, 5.41).

(a) Use these two points to estimate the gradient of the curve at $x = 2$

(b) Explain why this is only an estimate.

1 **(a)** The line joining the two points is a chord, and its gradient approximates the gradient of the curve between them.

2 Change in $y = 5.41 - 4.61 = 0.80$

3 Change in $x = 2.1 - 1.9 = 0.2$

4 Gradient $= \dfrac{0.80}{0.2} = 4$

5 **(b)** The gradient of a curve changes from point to point, so the chord's gradient is only an average over that small interval, not the exact gradient at $x = 2$ (although here the chord is short, so the estimate is good).

> **Answer:** (a) 4 (b) a chord gives an average gradient over an interval, not the exact gradient at a single point

10 **[5 marks]**

The graph of $y = x^2 - 3x$ has been drawn.

(a) Explain which straight line should be drawn on the same axes to solve $x^2 - 3x = 2x - 4$

(b) Solve the equation algebraically.

(c) Write down the coordinates of the two points where the curve and the line would cross.

1 **(a)** The solutions are the x-values where the two sides of the equation are equal.

2 The left-hand side is the curve already drawn, so draw the line $y = 2x - 4$ and read off the x-coordinates where the graphs cross.

3 **(b)** $x^2 - 3x = 2x - 4$, so $x^2 - 5x + 4 = 0$

4 Factorise: $(x - 1)(x - 4) = 0$, so $x = 1$ or $x = 4$

5 **(c)** Substitute into the line $y = 2x - 4$.

6 $x = 1$ gives $y = -2$ and $x = 4$ gives $y = 4$, so the points are $(1, -2)$ and $(4, 4)$.

> **Answer:** (a) draw $y = 2x - 4$ (b) $x = 1$ or $x = 4$ (c) $(1, -2)$ and $(4, 4)$