Real-Life Graphs

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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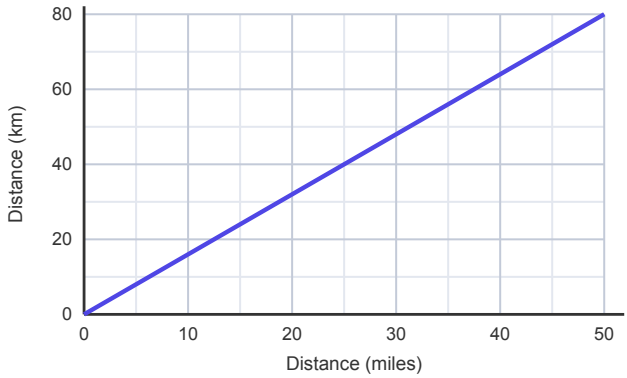
Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 **[1 mark]**

The conversion graph changes between miles and kilometres.



Use the graph to convert 20 miles into kilometres.

1 Find 20 on the horizontal (miles) axis and go straight up to the line.

2 Then read across to the vertical (kilometres) axis.

Answer: 32 km

2 **[2 marks]**

Use the same conversion graph to change 48 kilometres into miles.

1 This time start on the vertical axis at 48 km.

2 Go across to the line, then straight down to the miles axis.

3 This reads 30 miles. (Check: 30 × 1.6 = 48 ✓)

Answer: 30 miles

3 **[2 marks]**

The conversion line passes through (0, 0) and (50, 80).

(a) Work out the gradient of the line.

(b) What does the gradient tell you?

1 **(a)** Gradient $= \dfrac{80 - 0}{50 - 0} = 1.6$

2 **(b)** The gradient is the number of kilometres per mile.

3 So 1 mile is about 1.6 km.

Answer: (a) 1.6 (b) 1 mile ≈ 1.6 km

4 [2 marks]

A plumber charges a £40 call-out fee plus £25 for each hour worked.

Write a formula for the cost, C pounds, of a job lasting h hours.

1 The hourly part is 25 × h.

2 The call-out fee is added on whatever the length of the job.

> **Answer:** C = 25h + 40

5 [3 marks]

Using C = 25h + 40

(a) Work out the cost of a job lasting 3 hours.

(b) A job cost £165. How many hours did it take?

1 **(a)** C = 25 × 3 + 40 = 75 + 40 = £115

2 **(b)** 25h + 40 = 165

3 25h = 125, so h = 5 hours

> **Answer:** (a) £115 (b) 5 hours

6 [3 marks]

A cyclist travels 24 km in 1 hour 30 minutes.

(a) Work out the average speed in km/h.

(b) At the same speed, how long would it take to travel 40 km?

1 **(a)** 1 hour 30 minutes = 1.5 hours

2 Speed = $\frac{24}{1.5}$ = 16 km/h

3 **(b)** Time = $\frac{\text{distance}}{\text{speed}}$ = $\frac{40}{16}$ = 2.5 hours

4 That is 2 hours 30 minutes.

> **Answer:** (a) 16 km/h (b) 2.5 hours

7 [3 marks]

On a distance–time graph, describe what each of these tells you about the journey.

(a) A horizontal section.

(b) A steeper section.

(c) A section sloping back down to zero.

1 **(a)** The distance is not changing, so the object is stationary (stopped).

2 **(b)** A steeper gradient means more distance in the same time, so a faster speed.

3 **(c)** The distance from the start is decreasing, so the object is travelling back towards its starting point.

> **Answer:** (a) stopped (b) travelling faster (c) returning to the start

8 [3 marks]

The exchange rate is £1 = €1.15

(a) Change £250 into euros.

(b) Change €69 into pounds.

1 **(a)** Pounds to euros means multiplying: 250 × 1.15 = 287.5

2 So £250 = €287.50

3 **(b)** Euros to pounds means dividing: 69 ÷ 1.15 = 60

4 So €69 = £60

Answer: (a) €287.50 (b) £60

9 [4 marks]

Two gyms offer different deals.

Gym A: £30 per month plus £2 for each visit.

Gym B: £10 per month plus £6 for each visit.

(a) Write a formula for the monthly cost of each gym for v visits.

(b) For how many visits do the two gyms cost the same?

(c) Which gym is cheaper for someone who visits 8 times a month?

1 **(a)** Gym A: C = 2v + 30 Gym B: C = 6v + 10

2 **(b)** Set them equal: 2v + 30 = 6v + 10

3 20 = 4v, so v = 5 visits

4 **(c)** Gym A: 2 × 8 + 30 = £46

5 Gym B: 6 × 8 + 10 = £58, so Gym A is cheaper by £12.

Answer: (a) A: 2v + 30, B: 6v + 10 (b) 5 visits (c) Gym A (£46 v £58)

10 [4 marks]

A tank is filled with water at a constant rate.

After 2 minutes it holds 30 litres. After 5 minutes it holds 75 litres.

(a) Work out the rate of filling, in litres per minute.

(b) How much water was in the tank at the start?

(c) How long does it take to reach 200 litres?

1 **(a)** The volume rises by 75 − 30 = 45 litres in 5 − 2 = 3 minutes.

2 Rate = 45 ÷ 3 = 15 litres per minute (this is the gradient of the graph).

3 **(b)** Working back from 2 minutes: 30 − 2 × 15 = 0

4 The tank started empty.

5 **(c)** Time = 200 ÷ 15 = 13.33… minutes

6 That is 13 minutes 20 seconds.

Answer: (a) 15 litres/min (b) it was empty (c) 13 minutes 20 seconds

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1 **[2 marks]**

A car accelerates uniformly from rest to 24 m/s in 8 seconds.

Work out the acceleration.

1 Acceleration is the gradient of a speed–time graph.

2 Change in speed = 24 − 0 = 24 m/s over 8 s.

3 24 ÷ 8 = 3

Answer: 3 m/s^2

2 **[2 marks]**

(a) What does the gradient represent on a distance–time graph?

(b) What does the area under the graph represent on a speed–time graph?

1 **(a)** Gradient = change in distance ÷ change in time, which is the speed.

2 **(b)** Speed × time gives distance, and that is exactly what the area under the graph works out.

Answer: (a) speed (b) distance travelled

3 **[3 marks]**

A train travels at a constant 18 m/s for 25 seconds, then decelerates uniformly to rest over the next 12 seconds.

Work out the total distance travelled.

1 Sketch the speed–time graph: a rectangle followed by a triangle.

2 Rectangle: 18 × 25 = 450 m

3 Triangle: $\frac{1}{2} \times 12 \times 18 = 108$ m

4 Total = 450 + 108 = 558 m

Answer: 558 m

4 **[3 marks]**

A driver covers 45 km at an average speed of 90 km/h, then 30 km at an average speed of 60 km/h.

Work out the average speed for the whole journey.

1 Find each time: 45 ÷ 90 = 0.5 hours and 30 ÷ 60 = 0.5 hours.

2 Total time = 1 hour and total distance = 45 + 30 = 75 km.

3 Average speed = $\frac{75}{1} = 75$ km/h

4 (Note this is not the same as averaging 90 and 60, which would give 75 only by coincidence because the two times happen to be equal.)

Answer: 75 km/h

5 [4 marks]

The speed of a car, in m/s, is recorded every 2 seconds.

Time (s)	0	2	4	6
Speed (m/s)	0	5	12	21

(a) Use three trapezia to estimate the distance travelled in the first 6 seconds.

(b) Is your answer an under-estimate or an over-estimate? Explain.

1 **(a)** Each trapezium has width 2 and area $\frac{1}{2}$ × width × (sum of the two heights).

2 0 to 2 s: $\frac{1}{2} \times 2 \times (0 + 5) = 5$ m

3 2 to 4 s: $\frac{1}{2} \times 2 \times (5 + 12) = 17$ m

4 4 to 6 s: $\frac{1}{2} \times 2 \times (12 + 21) = 33$ m

5 Total = 5 + 17 + 33 = 55 m

6 **(b)** The speed is increasing faster and faster (gaps of 5, 7, 9), so the curve bends upwards and each straight chord lies above it — the estimate is too big.

Answer: (a) about 55 m (b) an over-estimate, because the curve bends upwards and the trapezia sit above it

6 [4 marks]

The depth of water, d cm, in a container is measured over time.

Time (min)	0	2	4	6
Depth (cm)	0	14	24	30

(a) Work out the average rate of increase between 2 and 4 minutes.

(b) Describe how the rate of filling changes, and suggest what this says about the shape of the container.

1 **(a)** Depth rises by 24 − 14 = 10 cm in 2 minutes.

2 Rate = 10 ÷ 2 = 5 cm per minute

3 **(b)** The rises in each 2-minute interval are 14 cm, 10 cm then 6 cm, so the depth is increasing more and more slowly.

4 If water flows in at a steady rate, the level rises more slowly where the container is wider, so the container must get wider towards the top.

Answer: (a) 5 cm/min (b) the rate is slowing down, so the container widens towards the top

7 [4 marks]

A cup of coffee cools so that its temperature T °C after t minutes is

$T = 20 + 60 \times 0.8^t$

(a) Work out the starting temperature.

(b) Work out the temperature after 5 minutes, to 1 decimal place.

(c) Explain what the number 20 represents.

1 **(a)** At t = 0, $0.8^0 = 1$, so T = 20 + 60 = 80 °C

2 **(b)** $0.8^5 = 0.32768$

3 T = 20 + 60 × 0.32768 = 20 + 19.6608 = 39.6608…

4 **(c)** As t grows, 0.8^t gets closer and closer to 0, so T approaches 20.

5 The 20 is the room temperature the coffee cools towards but never quite reaches.

Answer: (a) 80 °C (b) 39.7 °C (c) the room temperature it cools towards

8 [4 marks]

£2000 is invested at 5% compound interest per year.

(a) Work out the value after 10 years, to the nearest penny.

(b) A graph of value against time is drawn. Explain why it is a curve rather than a straight line.

1 **(a)** The multiplier is 1.05 each year, so the value is 2000×1.05^{10}

2 $1.05^{10} = 1.6288946…$

3 2000 × 1.6288946 = 3257.789…, so £3257.79

4 **(b)** With compound interest the interest each year is worked out on a larger amount than the year before.

5 So the yearly increase keeps growing, and a graph with an increasing gradient is a curve, not a straight line.

Answer: (a) £3257.79 (b) the yearly increase grows each year, so the gradient increases

9 [4 marks]

A car leaves home at 09:00, drives 90 km and arrives at 10:30. It stops for 30 minutes, then drives the 90 km home, arriving at 12:30.

(a) Work out the average speed on the outward journey.

(b) Work out the average speed on the return journey.

(c) Work out the average speed for the whole trip, including the stop, to 1 decimal place.

1 **(a)** 09:00 to 10:30 is 1.5 hours, so speed = 90 ÷ 1.5 = 60 km/h

2 **(b)** The stop ends at 11:00, so the return takes 11:00 to 12:30 = 1.5 hours.

3 Speed = 90 ÷ 1.5 = 60 km/h

4 **(c)** Total distance = 180 km and total time = 09:00 to 12:30 = 3.5 hours.

5 Average speed = $\frac{180}{3.5}$ = 51.428… km/h

Answer: (a) 60 km/h (b) 60 km/h (c) 51.4 km/h (1 d.p.)

10 **[5 marks]**

A train accelerates uniformly from rest to V m/s in 20 seconds. It then travels at V m/s for 60 seconds, before decelerating uniformly to rest in a further 40 seconds.

The total distance travelled is 3600 m.

(a) Find the value of V.

(b) Work out the acceleration during the first stage.

1 **(a)** The speed–time graph is a trapezium: triangle, rectangle, triangle.

2 First triangle: $\frac{1}{2} \times 20 \times V = 10V$

3 Rectangle: $60 \times V = 60V$ · Second triangle: $\frac{1}{2} \times 40 \times V = 20V$

4 Total area = $10V + 60V + 20V = 90V$, and this equals 3600

5 $V = 3600 \div 90 = 40$ m/s

6 **(b)** Acceleration = $\frac{40}{20} = 2$ m/s^2

Answer: (a) V = 40 m/s (b) 2 m/s^2