

Straight Line Graphs

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 [1 mark]

Write down the gradient of the line $y = 5x + 2$

1 In $y = mx + c$, the gradient is m — the number in front of x .

2 Here that number is 5, and the +2 is the y -intercept, not the gradient.

Answer: 5

2 [2 marks]

A line has equation $y = -3x + 7$

(a) Write down the gradient.

(b) Write down the coordinates of the y -intercept.

1 (a) The gradient is -3 , so the line slopes downwards.

2 (b) $c = 7$, and the y -intercept always has $x = 0$, giving $(0, 7)$

Answer: (a) -3 (b) $(0, 7)$

3 [2 marks]

Complete the table of values for $y = x + 3$

x	-2	0	2
y			

1 $x = -2$: $y = -2 + 3 = 1$

2 $x = 0$: $y = 0 + 3 = 3$

3 $x = 2$: $y = 2 + 3 = 5$

Answer: 1, 3, 5

4 [2 marks]

Which of these lines is parallel to $y = 4x - 1$?

A: $y = 4x + 3$ **B:** $y = -4x + 1$ **C:** $y = x + 4$

Give a reason for your answer.

1 Parallel lines have the same gradient.

2 The gradient of $y = 4x - 1$ is 4.

3 A has gradient 4, B has gradient -4 and C has gradient 1.

Answer: A — it has the same gradient of 4

5

[3 marks]

Work out the gradient of the line joining (2, 3) and (6, 11).

$$1 \quad \text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$$2 \quad \text{Change in } y = 11 - 3 = 8$$

$$3 \quad \text{Change in } x = 6 - 2 = 4$$

$$4 \quad \text{Gradient} = 8 \div 4 = 2$$

Answer: 2

6

[3 marks]

(a) Find the equation of the line with gradient 3 that passes through (0, -2).

(b) Show that the point (4, 10) lies on this line.

$$1 \quad \text{(a)} \text{ Use } y = mx + c \text{ with } m = 3.$$

$$2 \quad \text{The point } (0, -2) \text{ lies on the } y\text{-axis, so } c = -2, \text{ giving } y = 3x - 2$$

$$3 \quad \text{(b)} \text{ Substitute } x = 4: y = 3 \times 4 - 2 = 10$$

4 This matches the y -coordinate, so the point is on the line.

Answer: (a) $y = 3x - 2$ (b) $3(4) - 2 = 10 \checkmark$

7

[3 marks]

The line $2x + y = 8$ crosses the x -axis at A and the y -axis at B.

Write down the coordinates of A and B.

$$1 \quad \text{On the } x\text{-axis, } y = 0: 2x = 8, \text{ so } x = 4 \text{ and A is } (4, 0)$$

$$2 \quad \text{On the } y\text{-axis, } x = 0: y = 8, \text{ so B is } (0, 8)$$

3 A quick sketch confirms the line slopes downwards from (0, 8) to (4, 0).

Answer: A(4, 0) and B(0, 8)

8

[3 marks]

Rearrange $3y = 6x - 9$ into the form $y = mx + c$.

Write down the gradient and the y -intercept.

$$1 \quad \text{Divide every term by 3.}$$

$$2 \quad 3y \div 3 = y, \quad 6x \div 3 = 2x, \quad 9 \div 3 = 3$$

$$3 \quad y = 2x - 3$$

4 So the gradient is 2 and the y -intercept is (0, -3).

Answer: $y = 2x - 3$; gradient 2, y -intercept (0, -3)

9

[4 marks]

A straight line passes through (1, 4) and (3, 10).

Find the equation of the line.

$$1 \quad \text{Gradient} = \frac{10 - 4}{3 - 1} = \frac{6}{2} = 3$$

$$2 \quad \text{So the line is } y = 3x + c$$

$$3 \quad \text{Substitute (1, 4): } 4 = 3 \times 1 + c$$

$$4 \quad c = 1$$

$$5 \quad \text{The equation is } y = 3x + 1 \text{ (check with (3, 10): } 9 + 1 = 10 \checkmark).$$

Answer: $y = 3x + 1$

10

[4 marks]

A is the point (-1, 2) and B is the point (5, 10).

(a) Write down the coordinates of the midpoint of AB.

(b) Work out the gradient of AB.

(c) Work out the length of AB.

$$1 \quad \text{(a) Average the coordinates: } x = (-1 + 5) \div 2 = 2 \text{ and } y = (2 + 10) \div 2 = 6$$

$$2 \quad \text{The midpoint is (2, 6).}$$

$$3 \quad \text{(b) Gradient} = \frac{10 - 2}{5 - (-1)} = \frac{8}{6} = \frac{4}{3}$$

$$4 \quad \text{(c) Use Pythagoras on the horizontal and vertical differences, 6 and 8.}$$

$$5 \quad AB^2 = 6^2 + 8^2 = 36 + 64 = 100, \text{ so } AB = 10$$

Answer: (a) (2, 6) (b) $\frac{4}{3}$ (c) 10

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1

[2 marks]

Work out the gradient of the line joining (-3, 5) and (1, -3).

$$1 \quad \text{Change in } y = -3 - 5 = -8$$

$$2 \quad \text{Change in } x = 1 - (-3) = 4$$

$$3 \quad \text{Gradient} = -8 \div 4 = -2$$

Answer: -2

2

[2 marks]

Rearrange $4x - 2y = 10$ into the form $y = mx + c$.

$$1 \quad \text{Subtract } 4x \text{ from both sides: } -2y = -4x + 10$$

$$2 \quad \text{Divide every term by } -2, \text{ changing every sign.}$$

$$3 \quad y = 2x - 5$$

Answer: $y = 2x - 5$

3

[3 marks]

Find the equation of the line parallel to $y = -2x + 1$ that passes through the point (3, 1).

1 Parallel means the same gradient, so the line is $y = -2x + c$

2 Substitute (3, 1): $1 = -2 \times 3 + c = -6 + c$

3 $c = 7$

Answer: $y = -2x + 7$

4

[3 marks]

Find the equation of the line perpendicular to $y = \frac{1}{2}x - 4$ that passes through the point (2, 5).

1 Perpendicular gradients multiply to -1 .

2 $\frac{1}{2} \times m = -1$, so $m = -2$

3 The line is $y = -2x + c$, and substituting (2, 5) gives $5 = -4 + c$

4 $c = 9$

Answer: $y = -2x + 9$

5

[4 marks]

A is the point (1, 2) and B is the point (5, 10).

Find the equation of the perpendicular bisector of AB.

1 The perpendicular bisector goes through the midpoint of AB at right angles to it.

2 Midpoint = $((1 + 5) \div 2, (2 + 10) \div 2) = (3, 6)$

3 Gradient of AB = $\frac{8}{4} = 2$, so the perpendicular gradient is $-\frac{1}{2}$

4 The line is $y = -\frac{1}{2}x + c$

5 Substitute (3, 6): $6 = -1.5 + c$, so $c = 7.5$

Answer: $y = -\frac{1}{2}x + 7.5$ (or $x + 2y = 15$)

6

[4 marks]

A is $(-2, -5)$, B is $(1, 1)$ and C is $(4, 7)$.

(a) Show that A, B and C lie on the same straight line.

(b) Find the equation of that line.

1 (a) Gradient AB = $\frac{1 - (-5)}{1 - (-2)} = \frac{6}{3} = 2$

2 Gradient BC = $\frac{7 - 1}{4 - 1} = \frac{6}{3} = 2$

3 The gradients are equal and the two segments share the point B, so all three points lie on one line.

4 (b) Using $y = 2x + c$ with the point B(1, 1): $1 = 2 + c$, so $c = -1$

5 The line is $y = 2x - 1$ (check C: $2(4) - 1 = 7$ ✓).

Answer: (a) both gradients are 2 and they share B (b) $y = 2x - 1$

7

[4 marks]

The line L has equation $3x + 2y = 12$

- (a) Find the gradient of L.
- (b) Find the equation of the line perpendicular to L through (0, 1).
- (c) Does the point (6, -3) lie on L? Show your working.

1 (a) Rearrange: $2y = -3x + 12$, so $y = -\frac{3}{2}x + 6$

2 The gradient is $-\frac{3}{2}$

3 (b) The perpendicular gradient is $\frac{2}{3}$ (because $-\frac{3}{2} \times \frac{2}{3} = -1$).

4 The point (0, 1) gives the intercept directly: $y = \frac{2}{3}x + 1$

5 (c) Substitute into $3x + 2y$: $3(6) + 2(-3) = 18 - 6 = 12 \checkmark$

Answer: (a) $-\frac{3}{2}$ (b) $y = \frac{2}{3}x + 1$ (c) yes

8

[4 marks]

The line $y = -x + 6$ crosses the x-axis at P and the y-axis at Q. O is the origin.

- (a) Write down the coordinates of P and Q.
- (b) Find the exact length of PQ.
- (c) Find the area of triangle OPQ.

1 (a) $y = 0$ gives $x = 6$, so P is (6, 0). $x = 0$ gives $y = 6$, so Q is (0, 6).

2 (b) $PQ^2 = 6^2 + 6^2 = 72$

3 $PQ = \sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

4 (c) The triangle is right-angled at O with legs 6 and 6.

5 $\text{Area} = \frac{1}{2} \times 6 \times 6 = 18 \text{ square units}$

Answer: (a) P(6, 0), Q(0, 6) (b) $6\sqrt{2}$ (c) 18 square units

9

[4 marks]

The line through (2, k) and (5, 11) has gradient 3.

- (a) Find the value of k.
- (b) Find the equation of the line.

1 (a) Gradient = $\frac{11 - k}{5 - 2} = \frac{11 - k}{3}$

2 Set this equal to 3: $11 - k = 9$

3 $k = 2$

4 (b) Using $y = 3x + c$ with (5, 11): $11 = 15 + c$, so $c = -4$

5 The line is $y = 3x - 4$ (check (2, 2): $6 - 4 = 2 \checkmark$).

Answer: (a) $k = 2$ (b) $y = 3x - 4$

The line $y = 2x + 1$ meets the line $3x + 2y = 16$ at the point A.

(a) Find the coordinates of A.

(b) Find the equation of the line through A that is perpendicular to $y = 2x + 1$

(c) Find where this perpendicular line crosses the x -axis.

1 (a) Substitute $y = 2x + 1$ into $3x + 2y = 16$

2 $3x + 2(2x + 1) = 16$, so $7x + 2 = 16$ and $x = 2$

3 $y = 2(2) + 1 = 5$, so A is (2, 5).

4 (b) The perpendicular gradient is $-\frac{1}{2}$

5 $y = -\frac{1}{2}x + c$, and substituting A gives $5 = -1 + c$, so $c = 6$

6 The line is $y = -\frac{1}{2}x + 6$

7 (c) Set $y = 0$: $\frac{1}{2}x = 6$, so $x = 12$, giving (12, 0)

Answer: (a) (2, 5) (b) $y = -\frac{1}{2}x + 6$ (c) (12, 0)