

Fractions and Decimals

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 [1 mark]

Write $\frac{18}{24}$ in its simplest form.

1 The highest common factor of 18 and 24 is 6.

2 $18 \div 6 = 3$ and $24 \div 6 = 4$

Answer: $\frac{3}{4}$

2 [2 marks]

Work out $\frac{2}{5} + \frac{1}{3}$

1 You can only add fractions with the same denominator. The lowest common multiple of 5 and 3 is 15.

2 $\frac{2}{5} = \frac{6}{15}$ and $\frac{1}{3} = \frac{5}{15}$

3 Add the numerators only: $6 + 5 = 11$

Answer: $\frac{11}{15}$

3 [2 marks]

Work out $\frac{3}{4} \times \frac{2}{9}$

Give your answer in its simplest form.

1 To multiply fractions, multiply the tops and multiply the bottoms.

2 $3 \times 2 = 6$ and $4 \times 9 = 36$, giving $\frac{6}{36}$

3 Simplify by dividing top and bottom by 6.

Answer: $\frac{1}{6}$

4 [2 marks]

Write 0.6 as a fraction in its simplest form.

1 The 6 is in the tenths column, so $0.6 = \frac{6}{10}$

2 Divide top and bottom by 2.

Answer: $\frac{3}{5}$

5

[3 marks]

Work out $2\frac{1}{2} + 1\frac{3}{4}$

Give your answer as a mixed number.

1 Deal with the whole numbers first: $2 + 1 = 3$

2 Now the fractions: $\frac{1}{2} = \frac{2}{4}$, so $\frac{2}{4} + \frac{3}{4} = \frac{5}{4}$

3 $\frac{5}{4} = 1\frac{1}{4}$, so carry the extra whole one across.

4 $3 + 1\frac{1}{4} = 4\frac{1}{4}$

Answer: $4\frac{1}{4}$

6

[3 marks]

Work out $\frac{5}{8} \div \frac{3}{4}$

Give your answer in its simplest form.

1 To divide by a fraction, turn it upside down and multiply instead.

2 $\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3}$

3 Multiply across: $\frac{20}{24}$

4 Divide top and bottom by 4.

Answer: $\frac{5}{6}$

7

[3 marks]

(a) Work out $\frac{3}{5}$ of 45

(b) Work out $\frac{2}{7}$ of 91

1 'Of' means multiply. Divide by the bottom, then multiply by the top.

2 (a) $45 \div 5 = 9$, then $9 \times 3 = 27$

3 (b) $91 \div 7 = 13$, then $13 \times 2 = 26$

Answer: (a) 27 (b) 26

8

[3 marks]

Write these four numbers in order, starting with the smallest.

$$0.7 \quad \frac{2}{3} \quad 68\% \quad 0.71$$

1 Change everything into decimals so they can be compared fairly.

2 $\frac{2}{3} = 2 \div 3 = 0.666\dots$

3 $68\% = 68 \div 100 = 0.68$

4 Now compare 0.7, 0.666..., 0.68 and 0.71

5 Smallest first: 0.666..., 0.68, 0.7, 0.71

Answer: $\frac{2}{3}$, 68%, 0.7, 0.71

9

[4 marks]

A jug contains $\frac{3}{4}$ of a litre of juice.

Ali pours out $\frac{2}{5}$ of the juice.

How many litres of juice are left in the jug?

1 If $\frac{2}{5}$ is poured out, the fraction left is $1 - \frac{2}{5} = \frac{3}{5}$

2 So we need $\frac{3}{5}$ of $\frac{3}{4}$ of a litre.

3 $\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$

4 $\frac{9}{20}$ will not cancel, since 9 and 20 share no common factor.

5 As a decimal this is 0.45 litres, which is a sensible check — a bit more than half of 0.75.

Answer: $\frac{9}{20}$ of a litre (0.45 litres)

10

[4 marks]

A recipe uses $1\frac{1}{3}$ cups of flour for one batch of biscuits.

Kwame makes $4\frac{1}{2}$ batches.

How many cups of flour does he use?

1 Change both mixed numbers into improper fractions first.

2 $1\frac{1}{3} = \frac{4}{3}$ and $4\frac{1}{2} = \frac{9}{2}$

3 Multiply: $\frac{4}{3} \times \frac{9}{2} = \frac{36}{6}$

4 $36 \div 6 = 6$

5 Check roughly: about $1.33 \times 4.5 \approx 6$ ✓

Answer: 6 cups

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1 [2 marks]

Write the recurring decimal $0.363636\dots$ as a fraction in its simplest form.

1 Two digits repeat, so the fraction has 99 on the bottom: $\frac{36}{99}$

2 The highest common factor of 36 and 99 is 9.

3 $36 \div 9 = 4$ and $99 \div 9 = 11$

Answer: $\frac{4}{11}$

2 [2 marks]

Work out $1\frac{2}{3} \div 2\frac{1}{2}$

1 Convert to improper fractions: $1\frac{2}{3} = \frac{5}{3}$ and $2\frac{1}{2} = \frac{5}{2}$

2 Dividing means multiplying by the reciprocal: $\frac{5}{3} \times \frac{2}{5} = \frac{10}{15}$

3 Simplify by dividing by 5.

Answer: $\frac{2}{3}$

3 [3 marks]

Prove that the recurring decimal $0.4555\dots$ (only the 5 repeats) is equal to $\frac{41}{90}$

1 Let $x = 0.4555\dots$

2 Multiply by 10 so the repeating part starts straight after the point: $10x = 4.555\dots$

3 Multiply by 100: $100x = 45.555\dots$

4 Subtract the two lines — the endless 555... tails cancel: $100x - 10x = 45.555\dots - 4.555\dots$

5 $90x = 41$, so $x = \frac{41}{90}$ as required.

Answer: $0.4555\dots = \frac{41}{90}$

4 [3 marks]

Work out the value of $(\frac{2}{3})^{-2}$

1 A negative index means 'turn the fraction upside down'.

2 $(\frac{2}{3})^{-2} = (\frac{3}{2})^2$

3 Square top and bottom: $\frac{3^2}{2^2} = \frac{9}{4}$

Answer: $\frac{9}{4}$ (or $2\frac{1}{4}$)

5

[4 marks]

In a school, $\frac{3}{5}$ of the students are girls.

$\frac{2}{9}$ of the girls play hockey.

(a) What fraction of all the students are girls who play hockey?

(b) The school has 450 students. How many girls play hockey?

1 (a) ' $\frac{2}{9}$ of $\frac{3}{5}$ ' means multiply the fractions.

2 $\frac{2}{9} \times \frac{3}{5} = \frac{6}{45}$

3 Simplify by dividing top and bottom by 3: $\frac{2}{15}$

4 (b) $\frac{2}{15}$ of 450: $450 \div 15 = 30$, then $30 \times 2 = 60$

Answer: (a) $\frac{2}{15}$ (b) 60 girls

6

[4 marks]

Write as a single fraction in its simplest form

$$\frac{2}{x+1} + \frac{3}{x-2}$$

1 The common denominator is the product of the two brackets: $(x+1)(x-2)$

2 Rewrite each fraction: $\frac{2(x-2)}{(x+1)(x-2)} + \frac{3(x+1)}{(x+1)(x-2)}$

3 Expand the numerators: $2x - 4$ and $3x + 3$

4 Add them: $2x - 4 + 3x + 3 = 5x - 1$

5 Nothing cancels, so leave the bottom in factorised form.

Answer: $\frac{5x-1}{(x+1)(x-2)}$

7

[4 marks]

Work out $(2\frac{1}{4} - 1\frac{5}{6}) \div \frac{5}{6}$

1 Do the bracket first. $2\frac{1}{4} = \frac{9}{4}$ and $1\frac{5}{6} = \frac{11}{6}$

2 The lowest common denominator of 4 and 6 is 12: $\frac{27}{12} - \frac{22}{12} = \frac{5}{12}$

3 Now divide: $\frac{5}{12} \div \frac{5}{6} = \frac{5}{12} \times \frac{6}{5}$

4 $= \frac{30}{60} = \frac{1}{2}$

Answer: $\frac{1}{2}$

8

[4 marks]

Mia spends $\frac{2}{5}$ of her money on a coat.

She then spends $\frac{1}{3}$ of what is left on a pair of shoes.

Mia now has £36 left.

How much money did she start with?

1 After the coat she has $1 - \frac{2}{5} = \frac{3}{5}$ of her money left.

2 She spends $\frac{1}{3}$ of that, so $\frac{2}{3}$ of it remains.

3 Fraction still left = $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$

4 So $\frac{2}{5}$ of the original amount is £36.

5 $\frac{1}{5} = 36 \div 2 = £18$, so the whole amount is $5 \times 18 = £90$

6 Check: coat £36, leaving £54; shoes $\frac{1}{3}$ of 54 = £18, leaving £36 ✓

Answer: £90

9

[4 marks]

Simplify fully

$$\frac{x^2 - 4}{x^2 + 5x + 6} \times \frac{x + 3}{x - 2}$$

1 Factorise everything you can before multiplying.

2 $x^2 - 4 = (x + 2)(x - 2)$ (difference of two squares)

3 $x^2 + 5x + 6 = (x + 2)(x + 3)$

4 The product is $\frac{(x + 2)(x - 2)(x + 3)}{(x + 2)(x + 3)(x - 2)}$

5 Every factor on the top also appears on the bottom, so everything cancels.

Answer: 1

A rectangular garden measures $2\frac{3}{4}$ m by $1\frac{1}{3}$ m.

(a) Work out the area of the garden. Give your answer as a mixed number.

(b) Fencing costs £12 per metre. Work out the cost of fencing all the way round.

1 (a) Convert to improper fractions: $2\frac{3}{4} = \frac{11}{4}$ and $1\frac{1}{3} = \frac{4}{3}$

2 Area = $\frac{11}{4} \times \frac{4}{3} = \frac{44}{12} = \frac{11}{3} = 3\frac{2}{3} \text{ m}^2$

3 (b) Perimeter = $2 \times (\frac{11}{4} + \frac{4}{3})$

4 Common denominator 12: $\frac{33}{12} + \frac{16}{12} = \frac{49}{12}$

5 Perimeter = $2 \times \frac{49}{12} = \frac{49}{6} \text{ m}$

6 Cost = $\frac{49}{6} \times 12 = 49 \times 2 = \text{£}98$

Answer: (a) $3\frac{2}{3} \text{ m}^2$ (b) £98