

Number

GCSE Maths · Worked Solutions

20 questions · 53 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1

[1 mark]

Work out $7 + 2 \times 6$

1 BIDMAS tells you to do the multiplication before the addition.

2 $2 \times 6 = 12$

3 $7 + 12 = 19$

Answer: 19

2

[2 marks]

Write 45% as

(a) a decimal

(b) a fraction in its simplest form.

1 (a) Per cent means 'out of 100', so $45\% = \frac{45}{100} = 0.45$

2 (b) Start from $\frac{45}{100}$ and cancel.

3 The highest common factor of 45 and 100 is 5.

4 $45 \div 5 = 9$ and $100 \div 5 = 20$, giving $\frac{9}{20}$

Answer: (a) 0.45 (b) $\frac{9}{20}$

3

[2 marks]

Write down the value of

(a) 5^2

(b) $\sqrt{81}$

1 (a) 5^2 means $5 \times 5 = 25$

2 (b) $\sqrt{81}$ asks 'what number times itself gives 81?'

3 $9 \times 9 = 81$, so $\sqrt{81} = 9$

Answer: (a) 25 (b) 9

4

[2 marks]

Work out $\frac{3}{8}$ of 240

- 1 First find one eighth by dividing by the denominator.
- 2 $240 \div 8 = 30$
- 3 Now multiply by the numerator: $30 \times 3 = 90$

Answer: 90

5

[2 marks]

Write 90 as a product of its prime factors. Give your answer in index form.

- 1 Use a factor tree, dividing by the smallest prime each time.
- 2 $90 = 2 \times 45$
- 3 $45 = 3 \times 15$, and $15 = 3 \times 5$
- 4 So $90 = 2 \times 3 \times 3 \times 5$
- 5 In index form the two 3s become 3^2 .

Answer: $90 = 2 \times 3^2 \times 5$

6

[3 marks]

Work out $\frac{2}{3} + \frac{1}{4}$

Give your answer as a fraction in its simplest form.

- 1 You can only add fractions with the same denominator.
- 2 The lowest common multiple of 3 and 4 is 12, so write both over 12.
- 3 $\frac{2}{3} = \frac{8}{12}$ (multiply top and bottom by 4)
- 4 $\frac{1}{4} = \frac{3}{12}$ (multiply top and bottom by 3)
- 5 Now add the numerators only: $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$
- 6 11 and 12 share no common factor, so this is already simplest.

Answer: $\frac{11}{12}$

7

[3 marks]

A coat costs £80.

In a sale the price is reduced by 15%.

Work out the sale price of the coat.

- 1 *Method 1 — find the reduction, then subtract.*
- 2 10% of 80 = 8, so 5% of 80 = 4
- 3 15% of 80 = 8 + 4 = £12
- 4 Sale price = 80 – 12 = £68
- 5 *Method 2 — use a multiplier.* A 15% decrease means paying 85%, so multiply by 0.85.
- 6 $80 \times 0.85 = £68$ (same answer, and quicker)

Answer: £68

8

[3 marks]

(a) Write 0.0037 in standard form.

(b) Write 5.2×10^3 as an ordinary number.

1 (a) Standard form is $A \times 10^n$ where $1 \leq A < 10$.

2 Move the decimal point in 0.0037 three places right to get 3.7.

3 The number is smaller than 1, so the power is negative: 3.7×10^{-3}

4 (b) $10^3 = 1000$, so move the point three places right.

5 $5.2 \times 1000 = 5200$

Answer: (a) 3.7×10^{-3} (b) 5200

9

[4 marks]

Find

(a) the highest common factor (HCF) of 42 and 70

(b) the lowest common multiple (LCM) of 42 and 70.

1 Write each number as a product of primes first.

2 $42 = 2 \times 3 \times 7$

3 $70 = 2 \times 5 \times 7$

4 (a) For the HCF, multiply the primes that appear in *both* lists.

5 Both contain one 2 and one 7, so $\text{HCF} = 2 \times 7 = 14$

6 (b) For the LCM, take every prime that appears, to its highest power.

7 $\text{LCM} = 2 \times 3 \times 5 \times 7 = 210$

8 Check: $42 \times 70 = 2940$ and $14 \times 210 = 2940$ ✓ (HCF $\times$ LCM always equals the product of the two numbers)

Answer: (a) 14 (b) 210

10

[4 marks]

At a bus station, buses to Leeds leave every 12 minutes and buses to York leave every 18 minutes.

At 9 am a bus to Leeds and a bus to York leave at the same time.

(a) At what time do a Leeds bus and a York bus next leave together?

(b) Including the 9 am departure, how many times do they leave together up to and including 12 noon?

1 They leave together after a number of minutes that is a multiple of both 12 and 18 — so you need the LCM.

2 $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$

3 $\text{LCM} = 2^2 \times 3^2 = 36$ minutes

4 (a) 9:00 am + 36 minutes = **9:36 am**

5 (b) From 9 am to 12 noon is 3 hours = 180 minutes.

6 $180 \div 36 = 5$, so there are 5 further departures after 9 am.

7 Listing them: 9:00, 9:36, 10:12, 10:48, 11:24, 12:00

8 Including the 9 am one, that is 6 times in total.

Answer: (a) 9:36 am (b) 6 times

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1

[2 marks]

Evaluate

(a) 3^{-2}

(b) $16^{1/2}$

1 (a) A negative power means 'one over' the positive power.

2 $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

3 (b) A power of $\frac{1}{2}$ means the square root.

4 $16^{1/2} = \sqrt{16} = 4$

Answer: (a) $\frac{1}{9}$ (b) 4

2

[2 marks]

Simplify fully $\sqrt{72}$

1 Look for a factor of 72 that is a perfect square.

2 $72 = 36 \times 2$, and 36 is a square number.

3 $\sqrt{72} = \sqrt{36 \times 2}$

4 $\sqrt{36} = 6$, so $\sqrt{72} = 6\sqrt{2}$

Answer: $6\sqrt{2}$

3

[2 marks]

Work out $(4 \times 10^5) \times (3 \times 10^{-2})$

Give your answer in standard form.

1 Multiply the number parts and the powers of 10 separately.

2 $4 \times 3 = 12$

3 $10^5 \times 10^{-2} = 10^{5-2} = 10^3$ (add the indices)

4 This gives 12×10^3 , but 12 is not between 1 and 10.

5 $12 \times 10^3 = 1.2 \times 10^1 \times 10^3 = 1.2 \times 10^4$

Answer: 1.2×10^4

4

[2 marks]

Rationalise the denominator of $\frac{6}{\sqrt{3}}$ and simplify your answer.

- 1 Multiply the top and the bottom by $\sqrt{3}$. This does not change the value, because you are multiplying by 1.
- 2 $\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$
- 3 $\sqrt{3} \times \sqrt{3} = 3$, which is why the surd disappears from the bottom.
- 4 Now cancel: $6 \div 3 = 2$, giving $2\sqrt{3}$

Answer: $2\sqrt{3}$

5

[3 marks]

Write the recurring decimal 0.27 (that is, 0.272727...) as a fraction in its simplest form.

- 1 Let $x = 0.272727\dots$
- 2 Two digits repeat, so multiply by 100: $100x = 27.272727\dots$
- 3 Subtract the first line from the second — the recurring tails cancel exactly.
- 4 $100x - x = 27.2727\dots - 0.2727\dots$
- 5 $99x = 27$, so $x = \frac{27}{99}$
- 6 Cancel by 9: $\frac{27}{99} = \frac{3}{11}$

Answer: $\frac{3}{11}$

6

[3 marks]

Expand and simplify $(3 + \sqrt{5})(2 - \sqrt{5})$

- 1 Expand the brackets in the usual way (each term in the first times each in the second).
- 2 $3 \times 2 = 6$
- 3 $3 \times (-\sqrt{5}) = -3\sqrt{5}$
- 4 $\sqrt{5} \times 2 = 2\sqrt{5}$
- 5 $\sqrt{5} \times (-\sqrt{5}) = -5$ (because $\sqrt{5} \times \sqrt{5} = 5$)
- 6 Collect: $6 - 5 = 1$, and $-3\sqrt{5} + 2\sqrt{5} = -\sqrt{5}$

Answer: $1 - \sqrt{5}$

7

[3 marks]

The length of a rod is 12.4 cm, correct to 1 decimal place.

(a) Write down the error interval for the length.

(b) Four of these rods are joined to make a square. Work out the upper bound of the perimeter.

1 (a) Rounded to 1 d.p., so the rounding unit is 0.1 and half of it is 0.05.

2 Lower bound = $12.4 - 0.05 = 12.35$ cm

3 Upper bound = $12.4 + 0.05 = 12.45$ cm

4 Error interval: $12.35 \leq L < 12.45$

5 (b) For the largest possible perimeter, use the largest possible length.

6 Perimeter = $4 \times 12.45 = 49.8$ cm

Answer: (a) $12.35 \leq L < 12.45$ (b) 49.8 cm

8

[3 marks]

$x = 4.6$ and $y = 2.3$, each correct to 1 decimal place.

Work out the upper bound of $\frac{x}{y}$. Give your answer to 3 significant figures.

1 Write down the bounds for each value first.

2 $4.55 \leq x < 4.65$ and $2.25 \leq y < 2.35$

3 A fraction is largest when the top is as big as possible and the bottom as *small* as possible.

4 So use $\frac{4.65}{2.25}$

5 $4.65 \div 2.25 = 2.0666\dots$

6 To 3 significant figures this is 2.07

Answer: 2.07

9

[3 marks]

Evaluate $\frac{16}{81}^{-3/4}$

Give your answer as a fraction.

1 Deal with the negative sign first: it means turn the fraction upside down.

2 $\frac{16}{81}^{-3/4} = \frac{81}{16}^{3/4}$

3 The denominator of the power (4) means take the fourth root.

4 $\sqrt[4]{81} = 3$ and $\sqrt[4]{16} = 2$, giving $\frac{3}{2}$

5 The numerator of the power (3) means cube it.

6 $\frac{3}{2}^3 = \frac{27}{8}$

Answer: $\frac{27}{8}$ (= 3.375)

A car travels 96 metres, measured to the nearest metre, in 7.2 seconds, measured to 1 decimal place.

- (a) Calculate the upper bound for the average speed of the car. Give your answer to 3 significant figures.
 (b) Explain which bounds you used and why.

1 Find the bounds for each measurement.

2 Distance to the nearest metre: $95.5 \leq d < 96.5$

3 Time to 1 d.p.: $7.15 \leq t < 7.25$

4 $\text{Speed} = \frac{\text{distance}}{\text{time}}$, so the speed is greatest when the distance is greatest and the time is *smallest*.

5 (a) Upper bound = $\frac{96.5}{7.15}$

6 $96.5 \div 7.15 = 13.4965\dots$

7 To 3 significant figures this is 13.5 m/s

8 (b) The upper bound of the distance (96.5 m) was used because a longer distance in the same time means a faster speed. The lower bound of the time (7.15 s) was used because covering the distance in less time also means a faster speed. Together these give the largest possible value of the fraction.

Answer: (a) 13.5 m/s (b) greatest distance (96.5 m) $\div$ least time (7.15 s)