

Powers, Roots and Standard Form

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 [1 mark]

Work out 4^3

1 4^3 means $4 \times 4 \times 4$

2 $4 \times 4 = 16$, then $16 \times 4 = 64$

Answer: 64

2 [2 marks]

Work out

(a) $\sqrt{49}$

(b) the cube root of 27

1 (a) $7 \times 7 = 49$, so $\sqrt{49} = 7$

2 (b) $3 \times 3 \times 3 = 27$, so the cube root of 27 is 3

Answer: (a) 7 (b) 3

3 [2 marks]

Simplify $3^4 \times 3^2$, giving your answer as a single power of 3.

1 When multiplying powers of the same number, add the indices.

2 $4 + 2 = 6$

3 So the answer is 3^6 (which equals 729).

Answer: 3^6

4 [2 marks]

Write 5600 in standard form.

1 Standard form is $A \times 10^n$ where A is between 1 and 10.

2 Put the decimal point after the first digit: 5.6

3 The point has moved 3 places, so the power is 3.

Answer: 5.6×10^3

5

[3 marks]

(a) Write 0.00042 in standard form.

(b) Write 3.1×10^5 as an ordinary number.

1 (a) Move the point to just after the first non-zero digit: 4.2

2 The point moved 4 places to the right, so the index is negative: 4.2×10^{-4}

3 (b) A power of 5 means move the point 5 places to the right.

4 3.1 becomes 310 000

Answer: (a) 4.2×10^{-4} (b) 310 000

6

[3 marks]

(a) Work out $2^5 \div 2^2$

(b) Write down the value of 7^0

1 (a) When dividing powers of the same number, subtract the indices: $5 - 2 = 3$

2 $2^3 = 8$

3 (b) Anything (other than 0) raised to the power 0 equals 1.

Answer: (a) 8 (b) 1

7

[3 marks]

Work out $(3 \times 10^4) \times (2 \times 10^3)$

Give your answer in standard form.

1 Multiply the number parts and the powers separately.

2 $3 \times 2 = 6$

3 $10^4 \times 10^3 = 10^7$ (add the indices)

4 6 is between 1 and 10, so the answer is already in standard form.

Answer: 6×10^7

8

[3 marks]

In each pair, work out which is larger. Show your working.

(a) 2^6 or 3^4

(b) 5^3 or 2^7

1 (a) $2^6 = 64$ and $3^4 = 81$, so 3^4 is larger.

2 (b) $5^3 = 125$ and $2^7 = 128$, so 2^7 is larger.

3 Notice you cannot just compare the bases or the indices — you must work them out.

Answer: (a) $3^4 = 81$ (b) $2^7 = 128$

9

[4 marks]

- (a) Between which two consecutive whole numbers does $\sqrt{150}$ lie? Explain your answer.
 (b) By working out 12.2^2 and 12.3^2 , say which of these is closer to $\sqrt{150}$.

1 (a) $12^2 = 144$ and $13^2 = 169$

2 150 lies between 144 and 169, so $\sqrt{150}$ lies between 12 and 13.

3 (b) $12.2^2 = 148.84$ and $12.3^2 = 151.29$

4 $150 - 148.84 = 1.16$ while $151.29 - 150 = 1.29$

5 148.84 is the closer of the two, so $\sqrt{150}$ is a little nearer to 12.2

Answer: (a) between 12 and 13 (b) 12.2 is closer

10

[4 marks]

The mass of the Sun is 1.99×10^{30} kg.

The mass of the Earth is 5.97×10^{24} kg.

How many times heavier is the Sun than the Earth?

Give your answer in standard form to 3 significant figures.

1 Divide the Sun's mass by the Earth's mass.

2 Number parts: $1.99 \div 5.97 = 0.33333\dots$

3 Powers: $10^{30} \div 10^{24} = 10^6$ (subtract the indices)

4 So the answer is $0.3333\dots \times 10^6$

5 0.3333 is less than 1, so adjust: $0.3333 \times 10^6 = 3.333 \times 10^5$

Answer: 3.33×10^5 times heavier (3 s.f.)

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1

[2 marks]

Work out

(a) 5^{-2}

(b) $16^{3/4}$

1 (a) A negative index means 'one over': $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

2 (b) The bottom of the fraction index is a root and the top is a power.

3 The fourth root of 16 is 2, and $2^3 = 8$

Answer: (a) $\frac{1}{25}$ (b) 8

2

[2 marks]

Simplify $\sqrt{50} + \sqrt{8}$, giving your answer in the form $a\sqrt{2}$

1 Look for square factors. $50 = 25 \times 2$, so $\sqrt{50} = 5\sqrt{2}$

2 $8 = 4 \times 2$, so $\sqrt{8} = 2\sqrt{2}$

3 Now they are like terms: $5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$

Answer: $7\sqrt{2}$

3

[3 marks]

Rationalise the denominator of $\frac{10}{\sqrt{5}}$ and simplify your answer.

1 Multiply the top and the bottom by $\sqrt{5}$ — this does not change the value.

$$2 \quad \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5}$$

3 $\sqrt{5} \times \sqrt{5} = 5$, which clears the surd from the bottom.

4 $10 \div 5 = 2$, so the answer is $2\sqrt{5}$

Answer: $2\sqrt{5}$

4

[3 marks]

Expand and simplify $(3 + \sqrt{2})(4 - \sqrt{2})$

1 Multiply out all four pairs.

$$2 \quad 3 \times 4 = 12, \quad 3 \times (-\sqrt{2}) = -3\sqrt{2}$$

$$3 \quad \sqrt{2} \times 4 = 4\sqrt{2}, \quad \sqrt{2} \times (-\sqrt{2}) = -2$$

$$4 \quad \text{Collect: } 12 - 2 = 10 \text{ and } -3\sqrt{2} + 4\sqrt{2} = \sqrt{2}$$

Answer: $10 + \sqrt{2}$

5

[4 marks]

Work out $(4.8 \times 10^5) \div (1.6 \times 10^{-2})$

Give your answer in standard form.

1 Divide the number parts: $4.8 \div 1.6 = 3$

2 Divide the powers by subtracting the indices: $5 - (-2) = 5 + 2 = 7$

3 Take care with the double negative — subtracting -2 means adding 2.

4 So the answer is 3×10^7 , already in standard form.

Answer: 3×10^7

6

[4 marks]

Work out the value of $(\frac{8}{27})^{-2/3}$

- 1 The negative sign means turn the fraction over: $(\frac{27}{8})^{2/3}$
- 2 The 3 on the bottom of the index means take the cube root of top and bottom.
- 3 Cube root of 27 = 3 and cube root of 8 = 2, giving $\frac{3}{2}$
- 4 The 2 on the top of the index means square it: $(\frac{3}{2})^2 = \frac{9}{4}$

Answer: $\frac{9}{4}$ (or 2.25)

7

[4 marks]

Solve each equation.

(a) $2^x = 32$

(b) $3^{x+1} = 81$

(c) $5^x = \frac{1}{25}$

- 1 (a) Write 32 as a power of 2: $32 = 2^5$, so $x = 5$
- 2 (b) $81 = 3^4$, so $x + 1 = 4$
- 3 $x = 3$
- 4 (c) $\frac{1}{25} = 5^{-2}$, so $x = -2$

Answer: (a) 5 (b) 3 (c) -2

8

[4 marks]

A square has area 48 cm^2 .

(a) Find the exact length of one side, in the form $a\sqrt{b}$.

(b) Find the exact perimeter of the square.

- 1 (a) Side length = $\sqrt{48}$
- 2 Look for the largest square factor: $48 = 16 \times 3$
- 3 $\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3} \text{ cm}$
- 4 (b) Perimeter = $4 \times 4\sqrt{3} = 16\sqrt{3} \text{ cm}$
- 5 (As a check, $16\sqrt{3} \approx 27.7 \text{ cm}$, which is sensible for a square of area 48 cm^2 .)

Answer: (a) $4\sqrt{3} \text{ cm}$ (b) $16\sqrt{3} \text{ cm}$

9

[4 marks]

The Earth is approximately 1.5×10^8 km from the Sun.

Light travels at approximately 3×10^5 km per second.

Work out how many minutes light takes to travel from the Sun to the Earth. Give your answer to the nearest minute.

$$1 \quad \text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{1.5 \times 10^8}{3 \times 10^5}$$

$$2 \quad \text{Number parts: } 1.5 \div 3 = 0.5$$

$$3 \quad \text{Powers: } 10^8 \div 10^5 = 10^3$$

$$4 \quad \text{Time} = 0.5 \times 10^3 = 500 \text{ seconds}$$

$$5 \quad \text{Convert to minutes: } 500 \div 60 = 8.33... \text{ minutes}$$

Answer: about 8 minutes

10

[5 marks]

Rationalise the denominator of

$$\frac{5 + \sqrt{3}}{2 - \sqrt{3}}$$

Give your answer in the form $a + b\sqrt{3}$

$$1 \quad \text{Multiply the top and bottom by the **conjugate** of the bottom, which is } 2 + \sqrt{3}$$

$$2 \quad \text{Bottom: } (2 - \sqrt{3})(2 + \sqrt{3}) = 4 + 2\sqrt{3} - 2\sqrt{3} - 3 = 4 - 3 = 1$$

$$3 \quad \text{The surd terms cancel — that is the point of using the conjugate.}$$

$$4 \quad \text{Top: } (5 + \sqrt{3})(2 + \sqrt{3}) = 10 + 5\sqrt{3} + 2\sqrt{3} + 3$$

$$5 \quad = 13 + 7\sqrt{3}$$

$$6 \quad \text{So the fraction is } \frac{13 + 7\sqrt{3}}{1} = 13 + 7\sqrt{3}$$

Answer: $13 + 7\sqrt{3}$