

Rounding, Estimation and Bounds

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1

[1 mark]

Round 4.837 to 1 decimal place.

- 1 Look at the second decimal place, which is 3.
- 2 3 is less than 5, so the first decimal place stays as it is.

Answer: 4.8

2

[2 marks]

Round 27 483

- (a) to the nearest 100
- (b) to the nearest 1000

- 1 **(a)** The tens digit is 8, which is 5 or more, so round the hundreds up: 27 500
- 2 **(b)** The hundreds digit is 4, which is less than 5, so round down: 27 000

Answer: (a) 27 500 (b) 27 000

3

[2 marks]

Round 0.004617 to 2 significant figures.

- 1 Leading zeros do not count, so the first significant figure is the 4 and the second is the 6.
- 2 The next digit is 1, which is less than 5, so round down.
- 3 Keep the zeros that hold the place value.

Answer: 0.0046

4

[2 marks]

Round 78 452 to 3 significant figures.

- 1 The first three significant figures are 7, 8 and 4.
- 2 The next digit is 5, so round the 4 up to 5.
- 3 Replace the remaining digits with zeros to keep the size right.

Answer: 78 500

5 [3 marks]

Work out an estimate for 39.2×5.87

- 1 Round each number to 1 significant figure.
- 2 $39.2 \approx 40$ and $5.87 \approx 6$
- 3 $40 \times 6 = 240$
- 4 (The exact answer is 230.1..., so 240 is a reasonable estimate.)

Answer: about 240

6 [3 marks]

Work out an estimate for $\frac{28.7 \times 9.6}{4.9}$

- 1 Round each number to 1 significant figure.
- 2 $28.7 \approx 30$, $9.6 \approx 10$, $4.9 \approx 5$
- 3 Top: $30 \times 10 = 300$
- 4 $300 \div 5 = 60$

Answer: about 60

7 [3 marks]

A number, n , is rounded to the nearest 10. The result is 80.

- (a) Write down the smallest possible value of n .
- (b) Write down the error interval for n using inequalities.

- 1 Anything from 75 upwards rounds up to 80.
- 2 (a) The smallest value is 75
- 3 (b) At the top end, 85 would round up to 90, so 85 is not included.
- 4 The error interval is $75 \leq n < 85$

Answer: (a) 75 (b) $75 \leq n < 85$

8 [3 marks]

A length is measured as 6.4 cm, correct to 1 decimal place.

- (a) Write down the lower bound of the length.
- (b) Write down the upper bound of the length.

- 1 1 decimal place means the measurement is accurate to within half of 0.1, which is 0.05
- 2 (a) Lower bound = $6.4 - 0.05 = 6.35$ cm
- 3 (b) Upper bound = $6.4 + 0.05 = 6.45$ cm
- 4 (Strictly the true length can equal 6.35 but not 6.45.)

Answer: (a) 6.35 cm (b) 6.45 cm

9

[4 marks]

Work out an estimate for $\frac{4.87^2 \times 19.6}{0.51}$

- 1 Round each number to 1 significant figure.
- 2 $4.87 \approx 5$, $19.6 \approx 20$, $0.51 \approx 0.5$
- 3 Top: $5^2 \times 20 = 25 \times 20 = 500$
- 4 Dividing by 0.5 is the same as multiplying by 2.
- 5 $500 \div 0.5 = 1000$

Answer: about 1000

10

[4 marks]

Sam estimates 4.8×6.2 as $5 \times 6 = 30$

(a) Work out the exact value of 4.8×6.2

(b) Is Sam's estimate too big or too small? Explain your answer without using your answer to part (a).

- 1 (a) $4.8 \times 6.2 = 29.76$
- 2 (b) Sam rounded 4.8 *up* by 0.2 and rounded 6.2 *down* by 0.2.
- 3 Rounding 4.8 up to 5 adds roughly $6 \times 0.2 = 1.2$ to the answer.
- 4 Rounding 6.2 down to 6 takes away roughly $5 \times 0.2 = 1.0$
- 5 The increase is bigger than the decrease, so the estimate is too big — an overestimate.

Answer: (a) 29.76 (b) too big (an overestimate)

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1

[2 marks]

$x = 8.3$ correct to 1 decimal place.

Write down the error interval for x .

- 1 Half of the last place value is 0.05
- 2 Lower bound = $8.3 - 0.05 = 8.25$ and upper bound = $8.3 + 0.05 = 8.35$
- 3 8.35 would round up to 8.4, so it is not included.

Answer: $8.25 \leq x < 8.35$

2

[2 marks]

A mass m is 4.6 kg correct to 2 significant figures.

Write down the error interval for m .

- 1 2 significant figures here means the value is given to the nearest 0.1 kg.
- 2 Half of 0.1 is 0.05
- 3 So the bounds are $4.6 - 0.05 = 4.55$ and $4.6 + 0.05 = 4.65$

Answer: $4.55 \leq m < 4.65$

3

[3 marks]

$a = 12.4$ and $b = 3.7$, each correct to 1 decimal place.

Work out the upper bound of $a + b$.

- 1 Bounds for a : 12.35 to 12.45
- 2 Bounds for b : 3.65 to 3.75
- 3 To make a sum as large as possible, use the upper bound of both numbers.
- 4 $12.45 + 3.75 = 16.2$

Answer: 16.2

4

[3 marks]

$a = 12.4$ and $b = 3.7$, each correct to 1 decimal place.

Work out the lower bound of $a - b$.

- 1 To make a subtraction as small as possible, start from the smallest a and take away the largest b .
- 2 Lower bound of $a = 12.35$ and upper bound of $b = 3.75$
- 3 $12.35 - 3.75 = 8.6$
- 4 A common error is to use both lower bounds — that would give too large an answer.

Answer: 8.6

5

[4 marks]

A rectangle has length 8.4 cm and width 5.2 cm, each correct to 1 decimal place.

Work out the upper bound of the area of the rectangle.

- 1 Upper bound of the length = 8.45 cm
- 2 Upper bound of the width = 5.25 cm
- 3 For a product, the largest result comes from using both upper bounds.
- 4 $8.45 \times 5.25 = 44.3625$

Answer: 44.3625 cm²

6

[4 marks]

A runner covers 240 m, measured to the nearest 10 m, in 18 seconds, measured to the nearest second.

(a) Work out the upper bound of the average speed.

(b) Work out the lower bound of the average speed, to 2 decimal places.

- 1 Distance bounds: 235 m to 245 m. Time bounds: 17.5 s to 18.5 s.
- 2 (a) A fraction is largest when the top is largest and the bottom is smallest.
- 3 Upper bound of speed = $\frac{245}{17.5} = 14$ m/s
- 4 (b) Smallest top over largest bottom.
- 5 Lower bound of speed = $\frac{235}{18.5} = 12.7027... = 12.70$ m/s (2 d.p.)

Answer: (a) 14 m/s (b) 12.70 m/s

7

[4 marks]

A number x is written as 6.3

(a) Write the error interval if 6.3 is **rounded** to 1 decimal place.

(b) Write the error interval if 6.3 is **truncated** to 1 decimal place.

(c) Explain the difference between the two intervals.

1 (a) Rounding goes half a unit either side: $6.25 \leq x < 6.35$

2 (b) Truncating simply chops off the remaining digits, so anything from 6.3 up to (but not including) 6.4 truncates to 6.3

3 The interval is $6.3 \leq x < 6.4$

4 (c) A rounded value sits in the middle of its interval, while a truncated value sits at the bottom of its interval — truncating never gives a value below the stated one.

Answer: (a) $6.25 \leq x < 6.35$ (b) $6.3 \leq x < 6.4$ (c) rounding is centred on 6.3; truncating starts at 6.3

8

[4 marks]

A metal block has mass 250 g to the nearest 10 g and volume 40 cm^3 to the nearest cm^3 .

Work out the upper bound of the density, in g/cm^3 , to 2 decimal places.

1 Density = $\frac{\text{mass}}{\text{volume}}$

2 Mass bounds: 245 g to 255 g. Volume bounds: 39.5 cm^3 to 40.5 cm^3 .

3 For the largest density, take the largest mass and the smallest volume.

4 Upper bound = $\frac{255}{39.5} = 6.4556\dots$

Answer: 6.46 g/cm^3 (2 d.p.)

9

[4 marks]

$a = 6.4$ and $b = 2.7$, each correct to 1 decimal place.

(a) Work out the upper and lower bounds of $\frac{a}{b}$

(b) Hence write down the value of $\frac{a}{b}$ to a suitable degree of accuracy, explaining your answer.

1 Bounds for a : 6.35 to 6.45. Bounds for b : 2.65 to 2.75.

2 (a) Upper bound = $\frac{6.45}{2.65} = 2.4339\dots$

3 Lower bound = $\frac{6.35}{2.75} = 2.3090\dots$

4 (b) Compare the two bounds digit by digit: 2.43... and 2.30...

5 They agree on the leading 2 but disagree in the first decimal place.

6 So only the first significant figure is certain, and $\frac{a}{b} = 2$ to 1 s.f.

Answer: (a) upper 2.4339..., lower 2.3090... (b) 2 (1 s.f.), because the bounds only agree to 1 significant figure

A car travels 96.5 km, correct to the nearest 0.1 km, in 1.25 hours, correct to the nearest 0.01 hours.

- (a) Work out the upper bound of the average speed, to 2 decimal places.
- (b) Work out the lower bound of the average speed, to 2 decimal places.
- (c) Write the average speed to a suitable degree of accuracy, giving a reason.

1 Distance bounds: 96.45 km to 96.55 km. Time bounds: 1.245 h to 1.255 h.

2 (a) Largest distance $\div$ smallest time = $\frac{96.55}{1.245} = 77.5502\dots$ km/h

3 So the upper bound is 77.55 km/h (2 d.p.).

4 (b) Smallest distance $\div$ largest time = $\frac{96.45}{1.255} = 76.8525\dots$ km/h

5 So the lower bound is 76.85 km/h (2 d.p.).

6 (c) Rounding both bounds to 2 significant figures gives 78 and 77, which still disagree; rounding to 1 significant figure gives 80 for both.

7 So the speed can only safely be stated as 80 km/h (1 s.f.).

Answer: (a) 77.55 km/h (b) 76.85 km/h (c) 80 km/h (1 s.f.) — that is the most accurate value both bounds agree on