

Factorising and Solving Quadratics

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1

[1 mark]

Factorise $x^2 + 5x$

1 Both terms contain x , so take x outside a bracket.

2 $x^2 \div x = x$ and $5x \div x = 5$

Answer: $x(x + 5)$

2

[2 marks]

Factorise $x^2 + 7x + 10$

1 Find two numbers that multiply to 10 and add to 7.

2 $2 \times 5 = 10$ and $2 + 5 = 7$ ✓

Answer: $(x + 2)(x + 5)$

3

[2 marks]

Factorise $x^2 - 4x + 3$

1 The number term is positive and the x term is negative, so both numbers are negative.

2 $-1 \times -3 = 3$ and $-1 + (-3) = -4$ ✓

Answer: $(x - 1)(x - 3)$

4

[2 marks]

Solve $(x - 2)(x + 5) = 0$

1 If a product is zero, at least one factor must be zero.

2 $x - 2 = 0$ gives $x = 2$

3 $x + 5 = 0$ gives $x = -5$

Answer: $x = 2$ or $x = -5$

5

[3 marks]

Factorise $x^2 + x - 12$

1 The number term is negative, so the two numbers have different signs.

2 Pairs multiplying to -12 : 1 and -12 , 2 and -6 , 3 and -4 , 4 and -3 ...

3 $4 + (-3) = 1$ ✓

4 So the brackets are $(x + 4)(x - 3)$

Answer: $(x + 4)(x - 3)$

6

[3 marks]

Solve $x^2 + 6x + 8 = 0$

1 Two numbers multiplying to 8 and adding to 6 are 2 and 4.

2 $(x + 2)(x + 4) = 0$

3 $x = -2$ or $x = -4$

4 Check $x = -4$: $16 - 24 + 8 = 0$ ✓

Answer: $x = -2$ or $x = -4$

7

[3 marks]

Factorise

(a) $x^2 - 16$

(b) $x^2 - 100$

1 Both are a difference of two squares: $a^2 - b^2 = (a + b)(a - b)$

2 (a) $16 = 4^2$, so $x^2 - 16 = (x + 4)(x - 4)$

3 (b) $100 = 10^2$, so $x^2 - 100 = (x + 10)(x - 10)$

Answer: (a) $(x + 4)(x - 4)$ (b) $(x + 10)(x - 10)$

8

[3 marks]

Solve $x^2 - 5x = 0$

1 There is no number term, so take out the common factor x .

2 $x(x - 5) = 0$

3 $x = 0$ or $x - 5 = 0$

4 Do not divide both sides by x — that would lose the solution $x = 0$.

Answer: $x = 0$ or $x = 5$

9

[4 marks]

A rectangle has width x cm and length $(x + 2)$ cm. Its area is 48 cm^2 .

(a) Show that $x^2 + 2x - 48 = 0$

(b) Work out the length and width of the rectangle.

1 (a) Area = $x(x + 2) = x^2 + 2x$

2 Setting this equal to 48 and moving 48 across gives $x^2 + 2x - 48 = 0$

3 (b) Two numbers multiplying to -48 and adding to 2 are 8 and -6 .

4 $(x + 8)(x - 6) = 0$, so $x = -8$ or $x = 6$

5 A width cannot be negative, so $x = 6$ cm and the length is 8 cm ($6 \times 8 = 48$ ✓).

Answer: (b) width 6 cm, length 8 cm

10

[4 marks]

A curve has equation $y = x^2 - x - 6$

(a) Factorise $x^2 - x - 6$

(b) Write down where the curve crosses the x -axis.

(c) Write down where the curve crosses the y -axis.

(d) Write down the equation of the line of symmetry.

1 (a) Two numbers multiplying to -6 and adding to -1 are -3 and $+2$, so the answer is $(x - 3)(x + 2)$

2 (b) Set $y = 0$: $x = 3$ or $x = -2$, giving $(3, 0)$ and $(-2, 0)$

3 (c) Set $x = 0$: $y = -6$, giving $(0, -6)$

4 (d) The line of symmetry runs halfway between the two roots.

5 $(3 + (-2)) \div 2 = 0.5$, so the line is $x = 0.5$

Answer: (a) $(x - 3)(x + 2)$ (b) $(3, 0)$ and $(-2, 0)$ (c) $(0, -6)$ (d) $x = 0.5$

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1

[2 marks]

Factorise $2x^2 + 5x + 2$

1 Multiply the end numbers: $2 \times 2 = 4$. Two numbers multiplying to 4 and adding to 5 are 4 and 1.

2 $2x^2 + 4x + x + 2 = 2x(x + 2) + 1(x + 2)$

Answer: $(2x + 1)(x + 2)$

2

[2 marks]

Solve $x^2 - 11x + 30 = 0$

1 Two negative numbers multiplying to 30 and adding to -11 are -5 and -6 .

2 $(x - 5)(x - 6) = 0$

Answer: $x = 5$ or $x = 6$

3

[3 marks]

Solve $3x^2 - 12x = 0$

1 Take out the highest common factor, which is $3x$.

2 $3x(x - 4) = 0$

3 $3x = 0$ gives $x = 0$, and $x - 4 = 0$ gives $x = 4$

Answer: $x = 0$ or $x = 4$

4

[3 marks]

Solve $4x^2 - 25 = 0$

1 This is a difference of two squares: $4x^2 = (2x)^2$ and $25 = 5^2$

2 $(2x + 5)(2x - 5) = 0$

3 $2x = -5$ gives $x = -2.5$, and $2x = 5$ gives $x = 2.5$

Answer: $x = 2.5$ or $x = -2.5$

5

[4 marks]

Solve $2x^2 + 7x - 15 = 0$

- 1 Multiply the end numbers: $2 \times (-15) = -30$
- 2 Two numbers multiplying to -30 and adding to 7 are 10 and -3 .
- 3 Split the middle term: $2x^2 + 10x - 3x - 15 = 0$
- 4 Factorise in pairs: $2x(x + 5) - 3(x + 5) = 0$
- 5 $(2x - 3)(x + 5) = 0$, so $x = 1.5$ or $x = -5$

Answer: $x = 1.5$ or $x = -5$

6

[4 marks]

Solve $6x^2 - 11x + 3 = 0$

- 1 Multiply the end numbers: $6 \times 3 = 18$
- 2 Two numbers multiplying to 18 and adding to -11 are -9 and -2 .
- 3 $6x^2 - 9x - 2x + 3 = 0$
- 4 $3x(2x - 3) - 1(2x - 3) = 0$
- 5 $(3x - 1)(2x - 3) = 0$, so $x = \frac{1}{3}$ or $x = \frac{3}{2}$

Answer: $x = \frac{1}{3}$ or $x = \frac{3}{2}$

7

[4 marks]

Solve $(x + 2)(x - 1) = 18$

- 1 You cannot set each bracket equal to 18 — the product must equal **zero** first.
- 2 Expand: $x^2 + x - 2 = 18$
- 3 Rearrange: $x^2 + x - 20 = 0$
- 4 Factorise: $(x + 5)(x - 4) = 0$
- 5 $x = -5$ or $x = 4$. Check $x = 4$: $6 \times 3 = 18$ ✓

Answer: $x = 4$ or $x = -5$

8

[4 marks]

Solve $x + \frac{20}{x} = 9$

- 1 Multiply every term by x to clear the fraction: $x^2 + 20 = 9x$
- 2 Rearrange: $x^2 - 9x + 20 = 0$
- 3 Two numbers multiplying to 20 and adding to -9 are -4 and -5 .
- 4 $(x - 4)(x - 5) = 0$
- 5 Check $x = 4$: $4 + 5 = 9$ ✓

Answer: $x = 4$ or $x = 5$

9

[4 marks]

Solve $\frac{1}{x} + \frac{1}{x+1} = \frac{5}{6}$

- 1 Multiply every term by $6x(x+1)$ to clear all three fractions.
- 2 $6(x+1) + 6x = 5x(x+1)$
- 3 Expand: $6x + 6 + 6x = 5x^2 + 5x$
- 4 Rearrange: $5x^2 - 7x - 6 = 0$
- 5 $5 \times (-6) = -30$; two numbers multiplying to -30 and adding to -7 are -10 and 3 .
- 6 $5x(x-2) + 3(x-2) = 0$, so $(5x+3)(x-2) = 0$

Answer: $x = 2$ or $x = -\frac{3}{5}$

10

[5 marks]

A right-angled triangle has shorter sides of length x cm and $(x+7)$ cm, and hypotenuse $(x+8)$ cm.

- (a) Show that $x^2 - 2x - 15 = 0$
 - (b) Work out the lengths of all three sides.
- 1 **(a)** By Pythagoras: $x^2 + (x+7)^2 = (x+8)^2$
 - 2 $(x+7)^2 = x^2 + 14x + 49$ and $(x+8)^2 = x^2 + 16x + 64$
 - 3 $2x^2 + 14x + 49 = x^2 + 16x + 64$
 - 4 Subtract everything on the right: $x^2 - 2x - 15 = 0$ as required.
 - 5 **(b)** Factorise: $(x-5)(x+3) = 0$, so $x = 5$ or $x = -3$
 - 6 A length cannot be negative, so $x = 5$
 - 7 The sides are 5 cm, 12 cm and 13 cm (check: $25 + 144 = 169 \checkmark$).

Answer: (b) 5 cm, 12 cm and 13 cm