

The Quadratic Formula

GCSE Maths · Worked Solutions

20 questions · 62 marks total

jtmaths.org · free maths resources

Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 [1 mark]

For the equation $2x^2 + 5x - 3 = 0$, write down the values of a , b and c .

- 1 Compare with the standard form $ax^2 + bx + c = 0$
- 2 The sign in front of each term belongs to that number.

Answer: $a = 2$, $b = 5$, $c = -3$

2 [2 marks]

Work out the value of $b^2 - 4ac$ for $x^2 + 3x - 4 = 0$

- 1 Here $a = 1$, $b = 3$ and $c = -4$
- 2 $b^2 - 4ac = 3^2 - 4 \times 1 \times (-4)$
- 3 $= 9 + 16 = 25$ (take care: subtracting a negative gives an addition).

Answer: 25

3 [2 marks]

Use the quadratic formula to solve $x^2 + 3x - 4 = 0$

- 1 From the previous question, $b^2 - 4ac = 25$ and $\sqrt{25} = 5$
- 2 $x = \frac{-3 \pm 5}{2}$
- 3 $x = \frac{2}{2} = 1$ or $x = \frac{-8}{2} = -4$

Answer: $x = 1$ or $x = -4$

4 [2 marks]

Work out $b^2 - 4ac$ for $2x^2 - 4x + 5 = 0$ and say what this tells you about the solutions.

- 1 $a = 2$, $b = -4$, $c = 5$
- 2 $b^2 - 4ac = (-4)^2 - 4 \times 2 \times 5 = 16 - 40 = -24$
- 3 You cannot take the square root of a negative number, so there are no real solutions.

Answer: -24 — the equation has no real solutions

5

[3 marks]

Use the quadratic formula to solve $x^2 - 6x + 5 = 0$

1 $a = 1, b = -6, c = 5$

2 $b^2 - 4ac = 36 - 20 = 16$, and $\sqrt{16} = 4$

3 $x = \frac{6 \pm 4}{2}$

4 $x = \frac{10}{2} = 5$ or $x = \frac{2}{2} = 1$

Answer: $x = 5$ or $x = 1$

6

[3 marks]

Solve $x^2 + 2x - 1 = 0$

Give your answers to 2 decimal places.

1 $a = 1, b = 2, c = -1$

2 $b^2 - 4ac = 4 + 4 = 8$, and $\sqrt{8} = 2.8284\dots$

3 $x = \frac{-2 \pm 2.8284}{2}$

4 $x = \frac{0.8284}{2} = 0.4142\dots$ or $x = \frac{-4.8284}{2} = -2.4142\dots$

Answer: $x = 0.41$ or $x = -2.41$ (2 d.p.)

7

[3 marks]

Solve $2x^2 + 5x + 1 = 0$

Give your answers to 2 decimal places.

1 $a = 2, b = 5, c = 1$

2 $b^2 - 4ac = 25 - 8 = 17$, and $\sqrt{17} = 4.1231\dots$

3 The denominator is $2a = 4$

4 $x = \frac{-5 + 4.1231}{4} = -0.2192\dots$ or $x = \frac{-5 - 4.1231}{4} = -2.2808\dots$

Answer: $x = -0.22$ or $x = -2.28$ (2 d.p.)

8

[3 marks]

Solve $x^2 - 4x - 3 = 0$, giving your answers in surd form.

1 $a = 1, b = -4, c = -3$

2 $b^2 - 4ac = 16 + 12 = 28$

3 $x = \frac{4 \pm \sqrt{28}}{2}$

4 Simplify the surd: $\sqrt{28} = \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$

5 $x = \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7}$ (divide every term by 2).

Answer: $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$

9

[4 marks]

Solve $3x^2 - 7x + 2 = 0$

1 $a = 3, b = -7, c = 2$

2 $b^2 - 4ac = 49 - 24 = 25$

3 $\sqrt{25} = 5$, so the answers will be exact.

4 $x = \frac{7 \pm 5}{6}$

5 $x = \frac{12}{6} = 2$ or $x = \frac{2}{6} = \frac{1}{3}$

Answer: $x = 2$ or $x = \frac{1}{3}$

10

[4 marks]

A ball is thrown upwards. Its height, h metres, after t seconds is given by $h = 20t - 5t^2$

(a) Find the two times when the ball is 15 m above the ground.

(b) Explain why there are two answers.

1 (a) Set $h = 15$: $20t - 5t^2 = 15$

2 Rearrange: $5t^2 - 20t + 15 = 0$, and divide by 5: $t^2 - 4t + 3 = 0$

3 $b^2 - 4ac = 16 - 12 = 4$, and $\sqrt{4} = 2$

4 $t = \frac{4 \pm 2}{2}$, so $t = 3$ or $t = 1$

5 (b) The ball passes 15 m on the way up (after 1 second) and again on the way down (after 3 seconds).

Answer: (a) $t = 1$ s and $t = 3$ s (b) once going up and once coming down

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1

[2 marks]

Solve $x^2 + 5x + 2 = 0$, giving your answers to 2 decimal places.

1 $a = 1, b = 5, c = 2$, so $b^2 - 4ac = 25 - 8 = 17$

2 $\sqrt{17} = 4.1231\dots$

3 $x = \frac{-5 \pm 4.1231}{2}$, giving $-0.4384\dots$ and $-4.5616\dots$

Answer: $x = -0.44$ or $x = -4.56$ (2 d.p.)

2

[2 marks]

Solve $2x^2 - 3x - 4 = 0$, giving your answers to 2 decimal places.

1 $a = 2, b = -3, c = -4$, so $b^2 - 4ac = 9 + 32 = 41$

2 $\sqrt{41} = 6.4031\dots$ and $2a = 4$

3 $x = \frac{3 \pm 6.4031}{4}$, giving 2.3508... and -0.8508...

Answer: $x = 2.35$ or $x = -0.85$ (2 d.p.)

3

[3 marks]

Solve $5x^2 + 2x - 2 = 0$, giving your answers in surd form as simply as possible.

1 $a = 5, b = 2, c = -2$

2 $b^2 - 4ac = 4 + 40 = 44$

3 $\sqrt{44} = \sqrt{4} \times \sqrt{11} = 2\sqrt{11}$

4 $x = \frac{-2 \pm 2\sqrt{11}}{10}$

5 Divide every term by 2: $x = \frac{-1 \pm \sqrt{11}}{5}$

Answer: $x = \frac{-1 + \sqrt{11}}{5}$ or $\frac{-1 - \sqrt{11}}{5}$

4

[3 marks]

Solve $3x^2 - 6x + 1 = 0$, giving your answers in surd form as simply as possible.

1 $a = 3, b = -6, c = 1$

2 $b^2 - 4ac = 36 - 12 = 24$, and $\sqrt{24} = 2\sqrt{6}$

3 $x = \frac{6 \pm 2\sqrt{6}}{6}$

4 Divide every term by 2: $x = \frac{3 \pm \sqrt{6}}{3}$

Answer: $x = \frac{3 + \sqrt{6}}{3}$ or $\frac{3 - \sqrt{6}}{3}$

5

[4 marks]

Use the discriminant to state how many real solutions each equation has.

(a) $x^2 - 6x + 9 = 0$

(b) $2x^2 + x + 3 = 0$

(c) $x^2 + 4x - 1 = 0$

1 The discriminant is $b^2 - 4ac$: positive means two roots, zero means one repeated root, negative means none.

2 (a) $36 - 36 = 0$, so there is exactly one (repeated) solution.

3 (b) $1 - 24 = -23$, which is negative, so there are no real solutions.

4 (c) $16 + 4 = 20$, which is positive, so there are two distinct real solutions.

Answer: (a) one repeated root (b) none (c) two

6

[4 marks]

The equation $x^2 + kx + 16 = 0$ has two equal roots.

Find the possible values of k .

1 Two equal roots means the discriminant is zero.

2 $a = 1$, $b = k$, $c = 16$, so $k^2 - 4 \times 1 \times 16 = 0$

3 $k^2 - 64 = 0$, so $k^2 = 64$

4 $k = 8$ or $k = -8$ — remember the negative root.

5 Check $k = -8$: $x^2 - 8x + 16 = (x - 4)^2$ ✓

Answer: $k = 8$ or $k = -8$

7

[4 marks]

The equation $x^2 + 6x + k = 0$ has two different real solutions.

Find the range of possible values of k .

1 Two different real solutions means the discriminant is strictly positive.

2 $a = 1$, $b = 6$, $c = k$

3 $b^2 - 4ac = 36 - 4k > 0$

4 $36 > 4k$, so $9 > k$

5 That is, $k < 9$ (if $k = 9$ the roots would be equal).

Answer: $k < 9$

8

[4 marks]

Solve $(2x - 1)(x + 3) = 5$

Give your answers to 2 decimal places.

1 Expand the left-hand side: $2x^2 + 6x - x - 3 = 2x^2 + 5x - 3$

2 Set equal to 5 and rearrange: $2x^2 + 5x - 8 = 0$

3 $a = 2$, $b = 5$, $c = -8$, so $b^2 - 4ac = 25 + 64 = 89$

4 $\sqrt{89} = 9.4340\dots$ and $2a = 4$

5 $x = \frac{-5 \pm 9.4340}{4}$, giving $1.1085\dots$ and $-3.6085\dots$

Answer: $x = 1.11$ or $x = -3.61$ (2 d.p.)

9

[4 marks]

A rectangle has width x cm and length $(x + 3)$ cm.

Its area is 15 cm^2 .

Work out the width of the rectangle, correct to 2 decimal places.

1 Area: $x(x + 3) = 15$, so $x^2 + 3x - 15 = 0$

2 $a = 1$, $b = 3$, $c = -15$, so $b^2 - 4ac = 9 + 60 = 69$

3 $\sqrt{69} = 8.3066\dots$

4 $x = \frac{-3 \pm 8.3066}{2}$, giving $2.6533\dots$ or $-5.6533\dots$

5 A width must be positive, so reject the negative answer.

Answer: 2.65 cm (2 d.p.)

10

[5 marks]

A stone is thrown from the top of a wall. Its height, h metres above the ground, t seconds later is given by

$$h = 1.5 + 12t - 5t^2$$

(a) Write down the height of the wall.

(b) Find the time when the stone hits the ground, to 2 decimal places.

1 (a) The stone leaves the wall at $t = 0$, when $h = 1.5$, so the wall is 1.5 m high.

2 (b) The stone hits the ground when $h = 0$: $1.5 + 12t - 5t^2 = 0$

3 Multiply by -1 and reorder: $5t^2 - 12t - 1.5 = 0$

4 $a = 5$, $b = -12$, $c = -1.5$, so $b^2 - 4ac = 144 + 30 = 174$

5 $\sqrt{174} = 13.1909\dots$ and $2a = 10$

6 $t = \frac{12 \pm 13.1909}{10}$, giving $2.5191\dots$ or $-0.1191\dots$

7 Time cannot be negative, so $t = 2.52$ seconds.

Answer: (a) 1.5 m (b) 2.52 seconds (2 d.p.)