

Quadratics

GCSE Maths · Worked Solutions

20 questions · 62 marks total

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Section A · Foundation

Questions 1–3 are the most straightforward; they get harder as you go on. Question 10 is the most challenging.

1 [1 mark]

Expand $x(x + 4)$

1 Multiply each term inside the bracket by x .

2 $x \times x = x^2$ and $x \times 4 = 4x$

Answer: $x^2 + 4x$

2 [2 marks]

Expand and simplify $(x + 2)(x + 5)$

1 Multiply every term in the first bracket by every term in the second.

2 $x \times x = x^2$, $x \times 5 = 5x$, $2 \times x = 2x$, $2 \times 5 = 10$

3 Collect the two middle terms: $5x + 2x = 7x$

Answer: $x^2 + 7x + 10$

3 [2 marks]

Factorise $x^2 + 6x + 8$

1 Look for two numbers that **multiply** to 8 and **add** to 6.

2 Pairs multiplying to 8: 1 and 8 (sum 9), 2 and 4 (sum 6) ✓

3 So the brackets are $(x + 2)(x + 4)$

Answer: $(x + 2)(x + 4)$

4 [2 marks]

Solve $(x - 3)(x + 7) = 0$

1 If two things multiply to give 0, at least one of them must be 0.

2 Either $x - 3 = 0$, which gives $x = 3$

3 Or $x + 7 = 0$, which gives $x = -7$

Answer: $x = 3$ or $x = -7$

5

[3 marks]

Solve $x^2 + 5x + 6 = 0$

1 Factorise first. Two numbers multiplying to 6 and adding to 5 are 2 and 3.

2 $(x + 2)(x + 3) = 0$

3 Set each bracket to zero: $x + 2 = 0$ gives $x = -2$

4 $x + 3 = 0$ gives $x = -3$

5 Check: $(-2)^2 + 5(-2) + 6 = 4 - 10 + 6 = 0 \checkmark$

Answer: $x = -2$ or $x = -3$

6

[3 marks]

Factorise

(a) $x^2 - 25$

(b) $x^2 - 3x - 10$

1 (a) There is no x term and $25 = 5^2$, so this is the difference of two squares.

2 $x^2 - 25 = (x + 5)(x - 5)$

3 (b) Two numbers multiplying to -10 and adding to -3 . The signs must be different.

4 $-5 \times 2 = -10$ and $-5 + 2 = -3 \checkmark$

5 So $x^2 - 3x - 10 = (x - 5)(x + 2)$

Answer: (a) $(x + 5)(x - 5)$ (b) $(x - 5)(x + 2)$

7

[3 marks]

Solve $x^2 - 7x + 12 = 0$

1 Two numbers multiplying to $+12$ and adding to -7 must both be negative.

2 $-3 \times -4 = 12$ and $-3 + (-4) = -7 \checkmark$

3 $(x - 3)(x - 4) = 0$

4 So $x = 3$ or $x = 4$

Answer: $x = 3$ or $x = 4$

8

[3 marks]

Solve $x^2 = 9x$

1 Do **not** divide both sides by x — that would throw away one of the solutions.

2 Move everything to one side: $x^2 - 9x = 0$

3 Both terms have a common factor of x : $x(x - 9) = 0$

4 So $x = 0$ or $x - 9 = 0$, giving $x = 9$

5 Check: $0^2 = 9 \times 0 \checkmark$ and $9^2 = 81 = 9 \times 9 \checkmark$

Answer: $x = 0$ or $x = 9$

9

[4 marks]

A rectangle has width x cm and length $(x + 5)$ cm.

The area of the rectangle is 36 cm^2 .

(a) Show that $x^2 + 5x - 36 = 0$

(b) Work out the length and width of the rectangle.

1 (a) Area = width $\times$ length = $x(x + 5) = x^2 + 5x$

2 This equals 36, so $x^2 + 5x = 36$, and moving 36 across gives $x^2 + 5x - 36 = 0$

3 (b) Two numbers multiplying to -36 and adding to 5 are 9 and -4 .

4 $(x + 9)(x - 4) = 0$, so $x = -9$ or $x = 4$

5 A width cannot be negative, so reject $x = -9$ and take $x = 4$

6 Width = 4 cm, length = $4 + 5 = 9$ cm (check: $4 \times 9 = 36$ ✓)

Answer: (b) width 4 cm, length 9 cm

10

[4 marks]

A curve has equation $y = x^2 - 2x - 8$

(a) Factorise $x^2 - 2x - 8$

(b) Write down the coordinates of the points where the curve crosses the x -axis.

(c) Write down the coordinates of the point where the curve crosses the y -axis.

1 (a) Two numbers multiplying to -8 and adding to -2 are -4 and $+2$.

2 So $x^2 - 2x - 8 = (x - 4)(x + 2)$

3 (b) The curve crosses the x -axis where $y = 0$, so solve $(x - 4)(x + 2) = 0$

4 $x = 4$ or $x = -2$, giving the points $(4, 0)$ and $(-2, 0)$

5 (c) The curve crosses the y -axis where $x = 0$.

6 $y = 0^2 - 2(0) - 8 = -8$, giving $(0, -8)$

Answer: (a) $(x - 4)(x + 2)$ (b) $(4, 0)$ and $(-2, 0)$ (c) $(0, -8)$

Section B · Higher

Higher-tier style. Again, questions 1–3 are the gentlest and question 10 is the hardest.

1

[2 marks]

Solve $x^2 + 3x - 28 = 0$

1 Two numbers multiplying to -28 and adding to $+3$ are $+7$ and -4 .

2 $(x + 7)(x - 4) = 0$

3 So $x = -7$ or $x = 4$

Answer: $x = -7$ or $x = 4$

2

[2 marks]

Write $x^2 + 8x + 3$ in the form $(x + a)^2 + b$

- 1 Halve the coefficient of x : $8 \div 2 = 4$, so start with $(x + 4)^2$
- 2 $(x + 4)^2 = x^2 + 8x + 16$, which is 16 too big, so subtract 16.
- 3 $x^2 + 8x + 3 = (x + 4)^2 - 16 + 3 = (x + 4)^2 - 13$

Answer: $(x + 4)^2 - 13$

3

[3 marks]

Solve $2x^2 - 5x - 3 = 0$

- 1 Multiply the outer numbers: $2 \times (-3) = -6$. Find two numbers multiplying to -6 and adding to -5 : that is -6 and $+1$.
- 2 Split the middle term: $2x^2 - 6x + x - 3 = 0$
- 3 Factorise in pairs: $2x(x - 3) + 1(x - 3) = 0$
- 4 So $(2x + 1)(x - 3) = 0$
- 5 $2x + 1 = 0$ gives $x = -\frac{1}{2}$, and $x - 3 = 0$ gives $x = 3$

Answer: $x = -\frac{1}{2}$ or $x = 3$

4

[3 marks]

Solve $x^2 + 4x - 6 = 0$

Give your answers correct to 2 decimal places.

- 1 This does not factorise, so use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- 2 Here $a = 1$, $b = 4$, $c = -6$
- 3 $b^2 - 4ac = 4^2 - 4 \times 1 \times (-6) = 16 + 24 = 40$
- 4 $x = \frac{-4 \pm \sqrt{40}}{2}$ and $\sqrt{40} = 6.3245\dots$
- 5 $x = \frac{-4 + 6.3245}{2} = 1.1622\dots$ or $x = \frac{-4 - 6.3245}{2} = -5.1622\dots$
- 6 Rounding each to 2 decimal places gives 1.16 and -5.16

Answer: $x = 1.16$ or $x = -5.16$ (2 d.p.)

5

[4 marks]

Write $2x^2 - 12x + 5$ in the form $a(x + b)^2 + c$

- 1 Take the 2 out of the first two terms only: $2(x^2 - 6x) + 5$
- 2 Complete the square inside the bracket: $x^2 - 6x = (x - 3)^2 - 9$
- 3 Substitute back: $2[(x - 3)^2 - 9] + 5$
- 4 Multiply the 2 through the square bracket: $2(x - 3)^2 - 18 + 5$
- 5 Simplify the numbers: $2(x - 3)^2 - 13$

Answer: $2(x - 3)^2 - 13$

6

[4 marks]

The equation $x^2 + kx + 9 = 0$ has exactly one solution.

Work out the possible values of k .

1 A quadratic has exactly one (repeated) solution when the discriminant $b^2 - 4ac = 0$

2 Here $a = 1$, $b = k$ and $c = 9$

3 So $k^2 - 4 \times 1 \times 9 = 0$, which gives $k^2 - 36 = 0$

4 $k^2 = 36$, so $k = 6$ or $k = -6$ (remember the negative root).

5 Check $k = 6$: $x^2 + 6x + 9 = (x + 3)^2$, a repeated root at $x = -3$ ✓

Answer: $k = 6$ or $k = -6$

7

[4 marks]

Solve $x + \frac{6}{x} = 5$

1 Clear the fraction by multiplying every term by x .

2 $x \times x + \frac{6}{x} \times x = 5 \times x$, giving $x^2 + 6 = 5x$

3 Rearrange into the standard form: $x^2 - 5x + 6 = 0$

4 Two numbers multiplying to 6 and adding to -5 are -2 and -3 .

5 $(x - 2)(x - 3) = 0$, so $x = 2$ or $x = 3$

6 Check $x = 2$: $2 + 6 \div 2 = 2 + 3 = 5$ ✓

Answer: $x = 2$ or $x = 3$

8

[4 marks]

A curve has equation $y = x^2 - 6x + 11$

(a) Write the equation in the form $(x + a)^2 + b$

(b) Write down the coordinates of the turning point of the curve.

(c) Explain how you know the curve never crosses the x -axis.

1 (a) Half of -6 is -3 , so start with $(x - 3)^2 = x^2 - 6x + 9$

2 That is 9 too big, so $y = (x - 3)^2 - 9 + 11 = (x - 3)^2 + 2$

3 (b) $(x - 3)^2$ is smallest when it equals 0, i.e. when $x = 3$, and then $y = 2$

4 The turning point is a minimum at $(3, 2)$.

5 (c) A square is never negative, so $(x - 3)^2 + 2$ is always at least 2.

6 Since y can never be 0, the curve never reaches the x -axis.

Answer: (a) $(x - 3)^2 + 2$ (b) minimum at $(3, 2)$ (c) the smallest value of y is 2, which is above the axis

9

[4 marks]

Solve the simultaneous equations

$$y = x^2 - 3x + 4$$

$$y = 2x - 2$$

1 Both expressions equal y , so set them equal to each other.

2 $x^2 - 3x + 4 = 2x - 2$

3 Move everything to one side: $x^2 - 5x + 6 = 0$

4 Factorise: $(x - 2)(x - 3) = 0$, so $x = 2$ or $x = 3$

5 Substitute into the simpler (linear) equation. When $x = 2$, $y = 2(2) - 2 = 2$

6 When $x = 3$, $y = 2(3) - 2 = 4$

Answer: $x = 2, y = 2$ and $x = 3, y = 4$

10

[5 marks]

A rectangular lawn measures 12 m by 8 m.

A path of constant width x metres is laid all the way around the outside of the lawn.

The total area of the lawn and the path together is 192 m^2 .

(a) Show that $x^2 + 10x - 24 = 0$

(b) Work out the width of the path.

1 (a) The path adds x to each side, so the outer rectangle is $(12 + 2x)$ m by $(8 + 2x)$ m.

2 Total area = $(12 + 2x)(8 + 2x) = 96 + 24x + 16x + 4x^2 = 4x^2 + 40x + 96$

3 Set this equal to 192: $4x^2 + 40x + 96 = 192$, so $4x^2 + 40x - 96 = 0$

4 Divide every term by 4: $x^2 + 10x - 24 = 0$ as required.

5 (b) Two numbers multiplying to -24 and adding to 10 are 12 and -2 , so $(x + 12)(x - 2) = 0$

6 $x = -12$ or $x = 2$. A width cannot be negative, so $x = 2$

7 Check: $16 \times 12 = 192 \text{ m}^2 \checkmark$

Answer: (b) the path is 2 m wide