



# GCSE Maths

Complete Revision Summary Sheets

Foundation & Higher Tier · All Topics

1. Number

2. Algebra

3. Ratio & Proportion

4. Geometry

5. Trigonometry

6. Graphs

7. Probability

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9. Further Algebra ★

★ Higher Tier content clearly labelled throughout

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# 1 • Number

Integers, fractions, decimals, percentages, powers, roots & standard form

F Foundation

H Higher

## TYPES OF NUMBER

- **Natural numbers:** 1, 2, 3, 4, ...
- **Integers:** ...-2, -1, 0, 1, 2, ...
- **Rational:** expressible as  $p/q$  ( $p, q$  integers)
- **Irrational:** cannot be written as fraction (e.g.  $\sqrt{2}$ ,  $\pi$ )
- **Prime:** exactly 2 factors: 1 and itself
- **Square numbers:** 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
- **Cube numbers:** 1, 8, 27, 64, 125

## FACTORS, MULTIPLES & PRIMES

- **HCF** (Highest Common Factor): largest number that divides both
- **LCM** (Lowest Common Multiple): smallest number both divide into
- **Prime factorisation:** write as product of primes

$$360 = 2^3 \times 3^2 \times 5$$

$$\text{HCF}(36, 48): 2^2 \times 3 = 12$$

$$\text{LCM}(4, 6): 12$$

## % FRACTIONS, DECIMALS & PERCENTAGES

Fraction  $\rightarrow$  Decimal: divide numerator  $\div$  denominator

Decimal  $\rightarrow$  %: multiply by 100

% of amount:  $(\% \div 100) \times \text{amount}$

% change:  $(\text{change} \div \text{original}) \times 100$

Reverse %:  $\text{amount} \div (1 \pm \%/100)$

### EXAMPLE - REVERSE %

After 20% increase, price = £144. Find original.

$$144 \div 1.20 = \boxed{\text{£120}}$$

## < POWERS, ROOTS & STANDARD FORM

$a^m \times a^n = a^{m+n}$  (multiply  $\rightarrow$  add powers)

$a^m \div a^n = a^{m-n}$  (divide  $\rightarrow$  subtract)

$(a^m)^n = a^{mn}$  (power of power  $\rightarrow$  multiply)

$$a^0 = 1 \qquad a^{-n} = 1/a^n$$

$$a^{(1/n)} = \sqrt[n]{a} \qquad a^{(m/n)} = (\sqrt[n]{a})^m$$

**Standard form:**  $A \times 10^n$  where  $1 \leq A < 10$

$$3500 = 3.5 \times 10^3$$

## √ SURD RULES

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\sqrt{a} \div \sqrt{b} = \sqrt{a/b}$$

$$(\sqrt{a})^2 = a$$

$$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$$

**Simplify:** find largest perfect-square factor

$$\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}$$

## ÷ RATIONALISING THE DENOMINATOR

Multiply top and bottom to remove surd from denominator.

$$1/\sqrt{3} \times \sqrt{3}/\sqrt{3} = \sqrt{3}/3$$

$$\begin{aligned} 5/(2+\sqrt{3}) \times (2-\sqrt{3})/(2-\sqrt{3}) &= \\ (10-5\sqrt{3})/(4-3) &= 10-5\sqrt{3} \end{aligned}$$

**WORKED EXAMPLE**

Simplify  $(3+\sqrt{5})(3-\sqrt{5}) = 9 - 5$

$$= 4$$

## || CALCULATOR &amp; ESTIMATION TIPS

- Round to 1 or 2 significant figures before estimating
- Significant figures (s.f.): count from first non-zero digit
- Decimal places (d.p.): count from decimal point
- Error intervals: if  $x = 3.4$  (1 d.p.)  $\rightarrow 3.35 \leq x < 3.45$



# 2 · Algebra

Expressions, equations, inequalities, factorising & sequences

F Foundation

H Higher

## EXPANDING BRACKETS

$$a(b + c) = ab + ac$$

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

### EXAMPLE

Expand  $(2x+3)(x-4)$   
 $= 2x^2 - 8x + 3x - 12$

$$= 2x^2 - 5x - 12$$

## FACTORISING

- **Common factor:**  $6x^2 + 9x = 3x(2x + 3)$
- **Difference of squares:**  $x^2 - 16 = (x+4)(x-4)$
- **Trinomial:**  $x^2 + 5x + 6 = (x+2)(x+3)$
- **Grouping:**  $ax + ay + bx + by = (a+b)(x+y)$

### EXAMPLE - FACTORISE $2x^2 + 7x + 3$

Find two numbers:  $2 \times 3 = 6$ , sum = 7  $\rightarrow$  1 and 6

$$2x^2 + x + 6x + 3 = x(2x+1) + 3(2x+1)$$

$$= (x+3)(2x+1)$$

## SOLVING EQUATIONS

- Do the same operation to both sides
- Collect like terms, then isolate the variable
- **Simultaneous:** elimination or substitution

$$3x + 5 = 20 \rightarrow x = 5$$

Sim. equations:

$$2x + y = 7$$

$$x + y = 4 \rightarrow x=3, y=1$$

## INEQUALITIES

- Solve like equations BUT flip sign when multiplying/dividing by negative
- Show on number line:  $\circ$  = open (not included),  $\bullet$  = closed (included)

$$3x - 2 > 7 \rightarrow 3x > 9 \rightarrow x > 3$$

$$-2x \leq 8 \rightarrow x \geq -4 \text{ (sign flips!)}$$

Integer solutions of  $-1 \leq x < 4$ : -1, 0, 1, 2, 3

## SEQUENCES

- **Arithmetic (linear):** constant difference  $d$
- **Geometric:** constant ratio  $r$
- **nth term (arithmetic):**  $a + (n-1)d$
- **Quadratic nth term:** contains  $n^2$  (Higher)

Sequence: 5, 8, 11, 14, ...  $d = 3$   
 nth term =  $3n + 2$   
 (Check:  $n=1 \rightarrow 5 \checkmark$ )

## FORMULAE & REARRANGING

- Substitute known values and solve for unknown
- Change subject: inverse operations step by step

$$v = u + at \rightarrow \text{make } t \text{ subject:}$$

$$v - u = at$$

$$t = (v - u) / a$$

### EXAMPLE - MAKE R SUBJECT

$$A = \pi r^2 \rightarrow r^2 = A/\pi$$

$$r = \sqrt{A/\pi}$$

**Key Identities to Memorise:**  $a^2 - b^2 = (a+b)(a-b)$  |  $(a \pm b)^2 = a^2 \pm 2ab + b^2$  | Sum of arithmetic sequence =  $n/2 \times$   
(first + last)



# 3 · Ratio & Proportion

Ratio, proportion, rates of change, speed-distance-time & percentage change

F Foundation

H Higher

## RATIO

- Simplify by dividing by HCF: 12:8 → 3:2
- **Sharing:** find total parts, then calculate each
- **Equivalent ratios:** multiply/divide all parts by same number
- **Map scales:** 1:50 000 means 1 cm = 500 m

### SHARE £240 IN RATIO 3:5

Total parts = 8 | 1 part = £30

£90 : £150

## PROPORTION

- **Unitary method:** find value of 1 unit first
- **Best value:** divide price by quantity to get price per unit
- **Exchange rates:** multiply or divide consistently

5 pens cost £3.50

1 pen = £0.70

8 pens = £5.60

## SPEED, DISTANCE, TIME

Speed =  
Distance ÷ Time  
Distance =  
Speed × Time  
Time = Distance  
÷ Speed

Units: km/h, m/s, mph. Convert carefully!

## DENSITY & PRESSURE

Density = Mass  
÷ Volume  
Pressure =  
Force ÷ Area

Same triangle method as SDT applies.

## % CHANGE & GROWTH

Multiplier for  
n% increase:  $1 + n/100$   
Multiplier for  
n% decrease:  $1 - n/100$   
Compound  
interest:  
 $A = P(1 + r/100)^n$

### ★ Higher Tier — Direct & Inverse Proportion

#### ∝ DIRECT PROPORTION

- $y \propto x$  means  $y = kx$
- $y \propto x^2$  means  $y = kx^2$
- $y \propto \sqrt{x}$  means  $y = k\sqrt{x}$
- **Find k:** substitute known values

#### EXAMPLE

$y \propto x^2$ ,  $y=12$  when  $x=2 \rightarrow k=3$

When  $x=5$ :  $y = 3 \times 25 = 75$

#### ÷ INVERSE PROPORTION

- $y \propto 1/x$  means  $y = k/x$
- $y \propto 1/x^2$  means  $y = k/x^2$
- As  $x$  increases,  $y$  decreases (and vice versa)

#### EXAMPLE

$y \propto 1/x$ ,  $y=6$  when  $x=4 \rightarrow k=24$

When  $x=8$ :  $y = 24/8 = 3$



# 4 · Geometry & Measures

Angles, shapes, area, volume, circles, transformations & vectors

F Foundation

H Higher

## ANGLE RULES

- Angles on a straight line =  $180^\circ$
- Angles around a point =  $360^\circ$
- Vertically opposite angles are equal
- Alternate angles (Z-angles) are equal
- Co-interior angles sum to  $180^\circ$
- Corresponding angles (F-angles) are equal

## POLYGONS

$$\begin{aligned} \text{Interior angle sum} &= (n-2) \times 180^\circ \\ \text{Each interior angle (regular)} &= (n-2) \times 180 \div n \\ \text{Each exterior angle (regular)} &= 360 \div n \\ \text{Interior} + \text{Exterior} &= 180^\circ \end{aligned}$$

Triangle =  $180^\circ$ , Quad =  $360^\circ$ , Pentagon =  $540^\circ$ , Hexagon =  $720^\circ$

## AREA & PERIMETER

Shape	Area	Perimeter
Rectangle	$l \times w$	$2(l + w)$
Triangle	$\frac{1}{2} \times b \times h$	sum of sides
Parallelogram	$b \times h$	$2(a + b)$
Trapezium	$\frac{1}{2}(a+b) \times h$	sum of sides
Circle	$\pi r^2$	$2\pi r$

## VOLUME & SURFACE AREA

Shape	Volume
Cuboid	$l \times w \times h$
Prism	Area of cross-section $\times$ length
Cylinder	$\pi r^2 h$
Cone	$\frac{1}{3} \pi r^2 h$
Sphere	$\frac{4}{3} \pi r^3$
Pyramid	$\frac{1}{3} \times \text{base area} \times h$

## TRANSFORMATIONS

- **Translation:** describe with vector  $(x/y)$
- **Reflection:** state mirror line equation
- **Rotation:** state centre, angle, direction
- **Enlargement:** state scale factor & centre

Enlargement with  $SF < 1$  = reduction. Negative SF = rotation through centre.

## CONGRUENCE & SIMILARITY

- **Congruent:** same shape and size (SSS, SAS, ASA, RHS)
- **Similar:** same shape, different size
- Length ratio =  $k \rightarrow$  Area ratio =  $k^2 \rightarrow$  Volume ratio =  $k^3$

Similar triangles: if sides in ratio 2:3  
Areas in ratio 4:9, Volumes in ratio 8:27

### ▮ CIRCLE THEOREMS

- Angle at centre =  $2 \times$  angle at circumference
- Angles in same segment are equal
- Angle in semicircle =  $90^\circ$
- Opposite angles in cyclic quadrilateral =  $180^\circ$
- Tangent meets radius at  $90^\circ$
- Two tangents from external point are equal
- Alternate segment theorem

### → VECTORS

Adding:  $a + b$  (tip-to-tail)  
Subtracting:  $a - b = a + (-b)$   
Magnitude:  $|a| = \sqrt{x^2 + y^2}$   
Scalar multiple:  $k(x/y) = (kx/ky)$

Parallel vectors:  $b = ka$  for some scalar  $k$ . Midpoint of  $AB = A + \frac{1}{2}AB$



# 5 · Trigonometry & Pythagoras

Pythagoras' theorem, SOHCAHTOA, sine & cosine rules

F Foundation

H Higher

## △ PYTHAGORAS' THEOREM

Only for **right-angled** triangles.

$$a^2 + b^2 = c^2 \quad (c = \text{hypotenuse})$$

$$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

### FIND HYPOTENUSE: LEGS 3, 4

$$c = \sqrt{9 + 16} = \sqrt{25}$$

$$c = 5$$

## SOH CAH TOA

Labels: **O**pposite, **A**djacent, **H**ypotenuse (relative to angle  $\theta$ )

$$\begin{aligned} \text{Sin } \theta &= O/H & \rightarrow & O = H \cdot \sin\theta, & H \\ &= O/\sin\theta \\ \text{Cos } \theta &= A/H & \rightarrow & A = H \cdot \cos\theta, & H \\ &= A/\cos\theta \\ \text{Tan } \theta &= O/A & \rightarrow & O = A \cdot \tan\theta, & A \\ &= O/\tan\theta \end{aligned}$$

To find angle: use inverse  $\sin^{-1}$ ,  $\cos^{-1}$ , or  $\tan^{-1}$

## EXACT TRIGONOMETRIC VALUES (MUST KNOW)

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
$0^\circ$	0	1	0
$30^\circ$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3} = \sqrt{3}/3$
$45^\circ$	$1/\sqrt{2} = \sqrt{2}/2$	$1/\sqrt{2} = \sqrt{2}/2$	1
$60^\circ$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$90^\circ$	1	0	undefined

## ★ Higher Tier — Sine Rule, Cosine Rule & Area Formula

### SINE RULE

Use when you know: **two sides & non-included angle**, or two angles & one side.

$$a/\sin A = b/\sin B = c/\sin C$$

(Flip for finding angles:)

$$\sin A/a = \sin B/b = \sin C/c$$

### FIND SIDE B: A=40°, B=75°, A=8

$$b/\sin 75^\circ = 8/\sin 40^\circ$$

$$b = 8 \times \sin 75^\circ / \sin 40^\circ \approx 12.0$$

### COSINE RULE & AREA

Use when you know: **two sides & included angle**, or all three sides.

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\cos A = (b^2 + c^2 - a^2) / 2bc$$

$$\text{Area} = \frac{1}{2}ab \cdot \sin C$$

### AREA: A=6, B=9, C=35°

$$\text{Area} = \frac{1}{2} \times 6 \times 9 \times \sin 35^\circ$$

$$\approx 15.5 \text{ cm}^2$$



# 6 · Graphs & Functions

Straight lines, curves, sketching, functions & graph transformations

F Foundation

H Higher

## STRAIGHT LINE GRAPHS

$y = mx + c$   
 $m = \text{gradient} = \text{rise/run} = (y_2 - y_1) / (x_2 - x_1)$   
 $c = \text{y-intercept}$   
 Parallel lines: same gradient  
 Perpendicular: gradients multiply to  $-1$

### LINE THROUGH (1,3) AND (3,7)

$$m = (7-3)/(3-1) = 4/2 = 2$$

$$y - 3 = 2(x - 1) \rightarrow \boxed{y = 2x + 1}$$

## READING GRAPHS

- **Distance-time:** gradient = speed
- **Velocity-time:** gradient = acceleration; area = distance
- **Real-life graphs:** identify intercepts and gradients in context
- **Conversion graphs:** read off and interpolate

## ★ Higher Tier — Quadratic, Cubic & Other Graphs

### U QUADRATIC GRAPHS: $Y = AX^2 + BX + C$

- $a > 0 \rightarrow$  U-shape;  $a < 0 \rightarrow$  n-shape
- Roots: where graph crosses x-axis ( $y=0$ )
- y-intercept: at  $x=0$ ,  $y=c$
- Line of symmetry:  $x = -b/(2a)$
- Vertex (turning point): substitute x back in

### ~ OTHER GRAPH SHAPES

Equation	Shape
$y = x^3$	Cubic (S-curve)
$y = 1/x$	Reciprocal (two curves)
$y = a^x$	Exponential ( $a > 1 =$ growth)
$y = \sin x$	Wave, period $360^\circ$
$y = \cos x$	Wave, period $360^\circ$

## ★ Higher Tier — Functions & Graph Transformations

### F FUNCTIONS

$f(x)$ : substitute  $x$  into expression  
 $fg(x)$ : apply  $g$  first, then  $f$   
 $f^{-1}(x)$ : inverse function (swap  $x$  and  $y$ , rearrange)  
 Domain: set of allowed inputs  
 Range: set of possible outputs

### ↔ GRAPH TRANSFORMATIONS

Transformation	Effect
$f(x) + a$	Translate up by $a$
$f(x + a)$	Translate left by $a$
$af(x)$	Stretch vertically $\times a$
$f(ax)$	Stretch horizontally $\times 1/a$
$-f(x)$	Reflect in x-axis
$f(-x)$	Reflect in y-axis



# 7 · Probability

Basic probability, combined events, tree diagrams & Venn diagrams

F Foundation

H Higher

## BASIC PROBABILITY

$P(\text{event}) = \text{number of favourable outcomes} / \text{total outcomes}$

$P(A') = 1 - P(A)$  (complement)

$0 \leq P(A) \leq 1$  always

$P(\text{certain event}) = 1$

$P(\text{impossible event}) = 0$

Relative frequency  $\approx$  probability (with large number of trials)

## × COMBINED EVENTS

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Mutually exclusive:  $P(A \text{ or } B) = P(A) + P(B)$

Independent:  $P(A \text{ and } B) = P(A) \times P(B)$

**P(RED) = 0.3, P(BLUE) = 0.5 (MUTUALLY EXCLUSIVE)**

$P(\text{Red or Blue}) = 0.3 + 0.5 = \mathbf{0.8}$

## TREE DIAGRAMS

- Multiply along branches (AND)
- Add end probabilities for same outcome (OR)
- All branches from a point must sum to 1
- Use for 2 or more events in sequence

**P(HH) FROM TWO FAIR COIN TOSSES**

$P(H) \times P(H) = \frac{1}{2} \times \frac{1}{2} = \mathbf{\frac{1}{4}}$

## VENN DIAGRAMS

- $A \cup B = A \text{ or } B$  (union)
- $A \cap B = A \text{ and } B$  (intersection)
- $A' = \text{not } A$  (complement)
- $\xi = \text{universal set (rectangle)}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

## ★ Higher Tier — Conditional Probability

## CONDITIONAL PROBABILITY

$P(A | B) = P(A \cap B) / P(B)$

"Probability of A given B has occurred"

Dependent events:  $P(A \text{ and } B) = P(A) \times P(B|A)$

### EXAMPLE

Bag: 4 red, 6 blue. Draw 2 without replacement.

$P(RR) = 4/10 \times 3/9 = 12/90 = \mathbf{2/15}$

## FREQUENCY TREES

- Like tree diagrams but with actual frequencies
- Each branch shows a count, not a probability
- Use to estimate probabilities from data

Estimated  $P(A) = \text{frequency of } A / \text{total frequency}$



# 8 · Statistics

Averages, spread, charts, scatter graphs & cumulative frequency

F Foundation

H Higher

## AVERAGES & SPREAD

Measure	How to find
<b>Mean</b>	Sum ÷ number of values
<b>Median</b>	Middle value (ordered list)
<b>Mode</b>	Most frequent value
<b>Range</b>	Max – Min
<b>IQR</b>	Q3 – Q1

For even-number dataset: median = average of two middle values

## MEAN FROM FREQUENCY TABLE

Mean =  $\frac{\Sigma(fx)}{\Sigma f}$   
 where  $f$  = frequency,  $x$  = value  
 Grouped data: use midpoints for  $x$   
 Modal class: class with highest frequency  
 Median class: where cumulative freq reaches  $n/2$

X: 2, 3, 4 | F: 3, 5, 2

$\Sigma fx = 6+15+8=29$  |  $\Sigma f=10$

Mean =  $29/10 =$  **2.9**

## TYPES OF CHART

Chart	Used for
Bar chart	Discrete or categorical data
Pie chart	Proportions (angles sum to 360°)
Histogram	Continuous grouped data (area = frequency)
Frequency polygon	Comparing distributions
Stem & leaf	Show all data values, retains detail

## SCATTER GRAPHS & CORRELATION

- **Positive correlation:** both variables increase
- **Negative correlation:** one increases, other decreases
- **No correlation:** no obvious pattern
- **Line of best fit:** drawn through the "middle" of points
- **Interpolation vs Extrapolation:** interpolation is more reliable
- Correlation ≠ causation!

### ★ Higher Tier — Cumulative Frequency & Box Plots

## CUMULATIVE FREQUENCY

- Running total of frequencies
- Plot at **upper class boundaries**
- S-shaped (ogive) curve
- Median = value at  $n/2$
- Q1 = value at  $n/4$ ; Q3 = value at  $3n/4$
- IQR = Q3 – Q1

## BOX PLOTS (BOX-AND-WHISKER)

- Shows: Min, Q1, Median, Q3, Max
- Box = IQR (middle 50% of data)
- Whiskers extend to Min and Max
- Compare median position and box width for distribution shape
- Skewed data: median closer to one end

## △ SAMPLING & DATA COLLECTION

- **Random sampling:** every member has equal chance of selection — reduces bias
- **Stratified sampling:** divide into groups, sample in proportion to group size
- **Systematic:** every nth person
- **Biased sample:** not representative of the population



# 9 · Further Algebra

Quadratics in depth, completing the square, algebraic fractions, proof & iteration

H Higher Tier Only

## THE QUADRATIC FORMULA

Solves  $ax^2 + bx + c = 0$  for any quadratic.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant:  $\Delta = b^2 - 4ac$   
 $\Delta > 0 \rightarrow 2$  real roots  
 $\Delta = 0 \rightarrow 1$  repeated root  
 $\Delta < 0 \rightarrow$  no real roots

### SOLVE $2x^2 - 5x - 3 = 0$

$$a=2, b=-5, c=-3 \mid \Delta = 25+24 = 49$$

$$x = \frac{5 \pm 7}{4} \rightarrow \boxed{x = 3 \text{ or } x = -\frac{1}{2}}$$

## COMPLETING THE SQUARE

$$ax^2 + bx + c \rightarrow a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

For  $x^2 + bx + c$ :

$$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

### COMPLETE THE SQUARE: $x^2 + 6x + 2$

$$(x + 3)^2 - 9 + 2$$

$$\boxed{(x + 3)^2 - 7}$$

Min value: -7 at  $x = -3$

## ALGEBRAIC FRACTIONS

Simplify: cancel common factors  
(factorise first!)

Add/subtract: find common denominator

Multiply: multiply tops, multiply bottoms

Divide: multiply by reciprocal

### SIMPLIFY $\frac{x^2-4}{x^2+4x+4}$

$$= \frac{(x+2)(x-2)}{(x+2)^2}$$

$$\boxed{= \frac{(x-2)}{(x+2)}}$$

## ITERATION

- Used to find approximate solutions to equations
- Rearrange to form  $x_{n+1} = f(x_n)$
- Start with initial estimate  $x_0$
- Repeat until values converge to required accuracy

$$\text{e.g. } x^3 - 2x - 3 = 0$$

$$x_{n+1} = (2x_n + 3)^{1/3}$$

$$x_0=1 \rightarrow x_1=1.71 \rightarrow x_2=1.83 \rightarrow \dots$$

## PROOF

- **Odd integer:**  $2n + 1$
- **Even integer:**  $2n$
- **Consecutive integers:**  $n, n+1, n+2$
- **Consecutive even:**  $2n, 2n+2, 2n+4$
- **Disproof:** find a counter-example

### PROVE: ODD + ODD = EVEN

$$(2m+1) + (2n+1) = 2m + 2n + 2 = 2(m+n+1)$$

This is even. ■

## ↕ SIMULTANEOUS EQUATIONS (QUADRATIC)

One linear, one quadratic — use substitution.

$$y = x + 1 \quad \text{and} \quad y = x^2$$

$$\text{Sub: } x^2 = x + 1$$

$$x^2 - x - 1 = 0 \rightarrow \text{solve with}$$

formula

$$Y = X+3, Y = X^2+X+1$$

$$x+3 = x^2+x+1 \rightarrow x^2-2 = 0$$

$$x = \pm\sqrt{2}$$

**Higher Tier Quick Summary:** Quadratic formula — memorise it! | Completing the square gives vertex of parabola | Algebraic fractions — always factorise before cancelling | Proof — use algebra with  $n$  to represent general integers